A HANDBOOK OF PHYSICS MEASUREMENTS

BY

ERVIN S. FERRY

IN COLLABORATION WITH

O. W. SILVEY, G. W. SHERMAN, Jr.
AND D. C. DUNCAN

VOL. II

VIBRATORY MOTION, SOUND, HEAT, ELECTRICITY AND MAGNETISM

FIRST EDITION

NEW YORK
JOHN WILEY & SONS, Inc.
London: CHAPMAN & HALL, Limited
1918

Stanhope Press
F. H. GILSON COMPANY
BOSTON, U.S.A.

PREFACE

In this present volume are given those measurements of quantities in vibratory motion, sound, heat, electricity and magnetism which experience has shown to be most available for college and industrial laboratories. In making the selection from the large number which have been tested under practical laboratory conditions, due consideration has been given to the particular determinations now demanded by science and industry, the degree of precision readily obtainable, and the time required for the performance of the experiment and the computation of the result.

In addition to the standard experiments in heat, the mechanical engineer will be especially interested in the methods for the determination of the economy effected by steam-pipe coverings, the thermal value of coal and the thermal value of gas.

All of the physics determinations in electricity and magnetism ordinarily made by students of electrical engineering are described in detail. In addition, considerable space has been devoted to work on damped vibration and harmonic wave analysis. This work is of great importance to the student of alternate current phenomena.

In many laboratories little attention is given to measurements in sound. The recent war, however, has emphasized so strongly the utility of such work, that no apology is made for the inclusion of a number of experiments in this subject.

As in the first volume, " Fundamental Measurements, Properties of Matter and Optics," each chapter consists of two parts. The first part includes definitions, a description of the apparatus, the general theory of the methods, and the derivation of the equations used in the determinations of the quantities considered in the chapter. In the second part of the chapter each determination is described in detail with respect to the theory and manipulation of the experiment, and the computation of the result.

E. S. F.

PURDUE UNIVERSITY, LAFAYETTE, INDIANA,
Dec. 2, 1918.

CONTENTS

VOLUME II

CHAPTER I

Vibratory Motion

CHAPTER II

Sound

CHAPTER III

HEAT

CHAPTER IV

Electricity and Magnetism

APPENDIX

TABLES

A HANDBOOK OF PHYSICS MEASUREMENTS

VOL. II

CHAPTER I

VIBRATORY MOTION

1. Definitions. — A motion which goes through a certain series of changes, first in one direction and then in the opposite direction, in either equal or nearly equal regularly recurring intervals of time is called *vibratory* or *oscillatory*. The displacements from the equilibrium position may be either linear or angular. A vibratory motion in a straight line is called *reciprocating*. The maximum displacement from the equilibrium position is called the *amplitude* of the vibration. A motion which goes through the same series of changes at regularly recurring intervals is called *periodic*. Not all periodic motions are vibratory, and not all vibratory motions are periodic. The time which elapses between two consecutive passages of an oscillating point, in the same direction, through any given point of its path, is called the *period* of the oscillation.

Simple harmonic motion of translation is that periodic reciprocating motion which has at every instant a linear acceleration which is directed toward the center of the path, and which varies directly with the distance of the moving body from that point. *Simple harmonic motion of rotation* is that periodic vibratory motion which has at every instant an angular acceleration directed toward a position of equilibrium, and which varies directly with the angular displacement of the body from that position.

A periodic disturbance which is handed on successively from one portion of a medium to another is called a *wave motion*. The sort of wave motion easiest to discuss is that due to a simple harmonic

vibration of the particles composing a medium such that each particle performs its vibration slightly later than the particles to one side of it, and slightly sooner than the particles to the other side of it. Such a wave is called a simple harmonic wave. Although a medium is necessary for the production and propagation of a wave, this medium need not be matter.

2. Simple Harmonic Angular Vibration. — If a body freely suspended by a vertical wire be rotated through a small angle about a line coincident with the axis of the wire, and be then released, the body will execute a series of angular vibrations about this axis. This is an example of the very important sort of motion called simple harmonic motion of rotation. Some of the properties of this motion will now be investigated.

In the case of the above vibrating system, it is found by experiment that at any instant there is developed in the wire a restoring torque having a moment L about the axis of rotation directly proportional to the angular displacement ϕ from the equilibrium position. If torques, angular displacements, and angular accelerations are called positive when directed away from the equilibrium position, it follows that if the wire is twisted through any small angle, and then released, the angular acceleration of the body's motion at any succeeding instant is given by the relation

$$\mathbf{a} = -k\phi, \tag{1}$$

where k is a constant depending upon the dimensions and material of the wire and the suspended body. A motion represented by this equation, that is, an oscillatory motion having at every instant an angular acceleration directed toward a position of equilibrium which varies directly with the angular displacement of the rotating body from that position, is called *simple harmonic motion of rotation.*

3. Value of the Angular Displacement. — We find, (98) Vol. 1, that when a wire of length l, and radius r, made of a material of simple rigidity μ, is twisted through an angle ϕ radians, there is developed a restoring torque

$$L = -\left(\frac{\mu\pi r^4}{2l}\right)\phi = -\beta\phi, \tag{2}$$

where β represents the constant quantity within the parenthesis.

Representing by w and K the angular speed and the moment of inertia of the vibrating system, respectively, we have for the angular acceleration at any instant

$$\mathbf{a} = \frac{dw}{dt} = \frac{d^2\phi}{dt^2} = \frac{L}{K} = -\frac{\beta}{K}\phi = -k\phi. \tag{3}$$

In order to find the value of the angular displacement ϕ at any time after leaving the position of equilibrium, it will be necessary to first find the angular speed at any time t and then by integration we can find the angular displacement ϕ at the same time t.

Since the angular speed

$$w = \frac{d\phi}{dt}$$

the above equation can be put into the form

$$2\,w\left(\frac{dw}{dt}\right) = [-2\,wk\phi] = -2\,k\phi\left(\frac{d\phi}{dt}\right).$$

Whence, by integration,

$$w^2 = -k\phi^2 + c_1. \tag{4}$$

The value of the constant of integration c_1 is now to be found. At any turning point $w = 0$. Denote the angular displacement from the position of equilibrium at this time, that is, the *amplitude* of the vibration, by the symbol Φ. At this instant

$$c_1 = k\Phi^2.$$

Consequently (4) becomes

$$w^2 = k\,(\Phi^2 - \phi^2).$$

Extracting the square root and remembering that $w = \dfrac{d\phi}{dt}$,

$$\frac{d\phi}{(\Phi^2 - \phi^2)^{\frac{1}{2}}} = \sqrt{k}\,dt.$$

Integrating,

$$\sin^{-1}\frac{\phi}{\Phi} + c_2 = t\sqrt{k}. \tag{5}$$

Since time is reckoned from the instant when the body starts from its position of equilibrium, it follows that when $\phi = 0$, $t = 0$. *At this instant* the constant of integration

$$c_2 = -\sin^{-1} 0 = -n\pi,$$

where n may be any whole number 0, 1, 2, 3, etc. On substituting the value of c_2 in (5), we obtain

$$\sin^{-1}\frac{\phi}{\Phi} = t\sqrt{k} + n\pi.$$

Whence,

$$\frac{\phi}{\Phi} = \sin(t\sqrt{k} + n\pi) = \sin t\sqrt{k}\cos n\pi + \cos t\sqrt{k}\sin n\pi.$$

But since $\sin n\pi = 0$ and $\cos n\pi = \pm 1$, we have

$$\phi = \pm\Phi \sin t\sqrt{k}, \tag{6}$$

where

$$k = \frac{\beta}{K} = \frac{\mu\pi r^4}{2\,lK}. \tag{7}$$

4. The Period of a Simple Harmonic Motion of Rotation. — The interval of time between two successive passages of a vibrating body, in the same direction, through a given point of its path is called the *period* of the vibration.

By differentiating (6) we find the angular velocity of a body moving with simple harmonic vibration of rotation to be

$$\frac{d\phi}{dt} = \pm\Phi\sqrt{k}\cos t\sqrt{k}. \tag{8}$$

If time be reckoned from the instant the body traverses the equilibrium position, then the velocity equals zero at the end of one-quarter of a period, at the end of three-quarters of a period, etc. Thus, denoting the period by T, the velocity will be zero when $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$, etc.

An inspection of the above equation shows that if the velocity equals zero when $t = \frac{T}{4}$, it will also equal zero when

$$t = \frac{T}{4} + \frac{\pi}{\sqrt{k}}, \quad \frac{T}{4} + \frac{2\pi}{\sqrt{k}}, \quad \frac{T}{4} + \frac{3\pi}{\sqrt{k}}, \text{ etc.}$$

Whence, the period of the angular vibration has the value

$$T = \frac{2\pi}{\sqrt{k}} \tag{9}$$

or, from (3),

$$T\left[= \frac{2\pi}{\sqrt{k}}\right] = 2\pi\sqrt{-\frac{\phi}{\mathrm{a}}} = 2\pi\sqrt{\frac{K}{\beta}}. \tag{10}$$

5. An equation of frequent use is one expressing the angular displacement in terms of the period of vibration T and the angular velocity w_0, when passing through the position of equilibrium. On substituting in (6) the value of $\sqrt{k}$ obtained from (9),

$$\phi = \pm\Phi \sin\frac{2\pi t}{T}. \tag{11}$$

When $t = 0$, the body is passing through the position of equilibrium. From (8) the angular velocity at this instant equals

$$w_0 = \pm\Phi\sqrt{k} \tag{12}$$

$$= \pm\Phi\frac{2\pi}{T}. \quad \text{[From (9).]} \tag{13}$$

Consequently,

$$\phi = \pm\frac{w_0 T}{2\pi}\sin\frac{2\pi t}{T}. \tag{14}$$

6. The Displacement of a Point Moving with Simple Harmonic Motion of Translation. — In elementary dynamics it is proved that "if a point moves with uniform speed in the circumference of a circle, the projection of the point on any straight line in the plane of the circle moves with simple harmonic motion of translation." Thus, in Fig. 1, if the point P moves with uniform speed in the circumference of the circle, the point p moves back

and forth along BB' with simple harmonic motion. The amplitude of the motion will equal the radius of the circle.

Let time and angular displacement be reckoned from the axis AA' inclined at the angle β to the axis XX'. Representing the amplitude by a, the linear displacement of p from the position of equilibrium O, at any time t, is

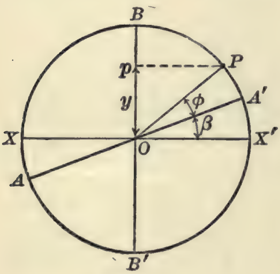

FIG. 1.

$$y = a \sin(\phi + \beta).$$

Since the speed of P in the circle is constant,

$$\frac{t}{T} = \frac{\phi}{2\pi},$$

where T represents the period, *i.e.*, the time of one complete vibration. Whence,

$$y\,[= a \sin(\phi + \beta)] = a \sin\left(\frac{2\pi t}{T} + \beta\right). \tag{15}$$

7. The Harmonic Curve. — The motion of a point compounded of a rectilinear simple harmonic motion and a uniform linear motion at right angles to the first is in a path called a harmonic curve. If the simple harmonic component be along the axis of ordinates, and the other be along the axis of abscissæ, we may write

$$y = a \sin\left(\frac{2\pi t}{T} + \beta\right)$$

and

$$x = st,$$

where s represents the uniform linear speed along the axis of abscissæ.

Eliminating t between these two equations,

$$y = a \sin\left(\frac{2\pi x}{sT} + \beta\right).$$

Representing the distance travelled in time T by λ,

$$s = \frac{\lambda}{T}.$$

Hence,

$$y = a \sin\left(\frac{2\pi x}{\lambda} + \beta\right). \tag{16}$$

8. Resultant of Two Simple Harmonic Motions Perpendicular to Each Other. Lissajous' Figures. — The path traversed by a point moving with the resultant of two simple harmonic motions at right angles to one another is called a Lissajous figure. These figures change markedly with changes in either the relative periods of the component vibrations, or the phase difference between the components. This fact is the basis of an important method of comparing the pitches of two sounding bodies.

The method for finding graphically the Lissajous figure for two components of any relative period and amplitude, and with any given phase difference between the components, will now be illustrated by a concrete example. Suppose that one component

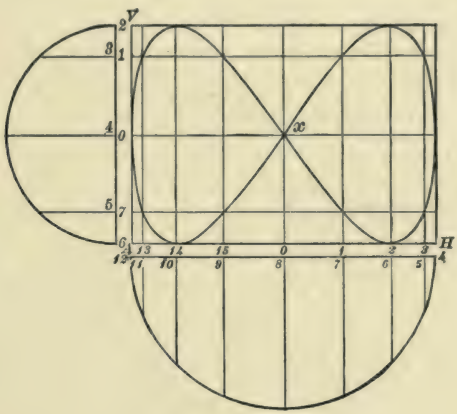

FIG. 2.

simple harmonic motion is vertical and that the other is horizontal; that the period of the horizontal component is twice that of the vertical; and that the amplitude of the horizontal component is 1.25 that of the vertical.

In Fig. 2, let the vertical component simple harmonic motion take place along the line AV, and the horizontal component along the line AH, the distance AH being (1.25) AV. In elementary dynamics it is proved that if a point moves with uniform speed in the circumference of a circle, the projection of the point on any

straight line in the plane of the circle moves with simple harmonic motion. From this fact we can find the distance travelled in equal times by a point moving along any axis with simple harmonic motion. Thus, remembering that in the present example the period of the horizontal component is twice that of the vertical component, we see that if we divide the semicircle described about AV into one-half as many equal arcs as we divide the semicircle described about AH, the projections of these arcs on AV and AH will be traversed by the vibrating points in equal times. For example, the points moving with simple harmonic motion along AV and AH traverse the points marked 0, 1, 2, 3, etc., at the ends of successive equal intervals of time. The phase of the vertical component vibration is 1/8 when the point vibrating along AV is traversing the point "1" (on AV), in the positive direction, and it is 3/8 when traversing the same point in the negative direction. Similarly, the phase of the horizontal component vibration is 1/8 when the point vibrating along AH is passing through the point "2" (on AH), in the positive direction, and it is 3/8 when passing through the same point in the negative direction.

Let it be required to construct the Lissajous figure when the horizontal component is (*a*) in phase with the vertical component; (*b*) one-eighth of a period in advance of the vertical component.

(*a*) *Horizontal Component in Phase with the Vertical Component.* — When the two points vibrating along AV and AH are in the equilibrium positions, the resultant is at the intersection x of the two lines perpendicular to AV and AH and passing through the points marked "0" in Fig. 2. At one interval of time later, the resultant is at the intersection of the two lines passing through the points "1." One equal interval later, the resultant is at the intersection of the lines through the points "2." The remainder of the Lissajous figure due to two vibrations of amplitudes in the ratio 1 : 1.25, and of periods in the ratio 1 : 2, the two components being in the same phase, is shown in Fig. 2.

(*b*) *Horizontal Component One-eighth of a Period in Advance of the Vertical Component.* — When the vertical component is in its equilibrium position and the horizontal component is one-eighth of a period in advance of its equilibrium position, the resultant is

at the intersection x, Fig. 3, of the two lines perpendicular to the vertical and the horizontal axes and passing through the points marked "0" and "2," on the respective axes. At one interval of time later, the resultant is at the intersection of the line perpendicular to the vertical axis at the point "1," and the line perpendicular to the horizontal axis at the point "3." One equal interval later, the resultant is at the intersection of the line perpendicular to the vertical axis at the point "2," and the line perpendicular to the horizontal axis at the point "4." One equal interval later, the resultant is at the intersection of the line perpendicular to the vertical axis at the point "3," and the line perpendicular to the horizontal axis at the point "5." In one complete period, the resultant traverses and retraces the curve shown in Fig. 3.

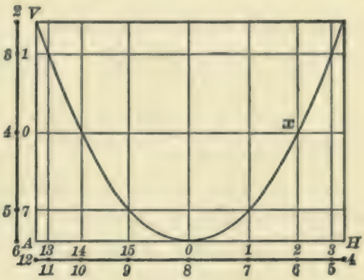

FIG. 3.

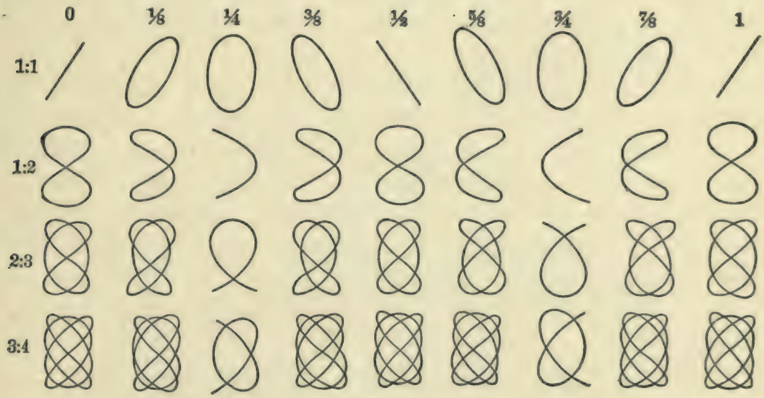

FIG. 4.

Lissajous' figures having components of various ratios of period and with various phase differences between the components are shown in Fig. 4. Each horizontal row shows figures due to com-

ponents having the period in the vertical direction and that in the horizontal direction in the ratio given at the left of the row. The number above each column expresses the phase which the horizontal component is in advance of the vertical component.

If one draws lines through any Lissajous figure parallel to the axes of the component vibrations, and notes the number of intersections which these lines make with the figure, one will observe that the ratio between the numbers of intersections made by the two lines equals the ratio between the periods of the components in those directions.

If x horizontal vibrations occur in the same time as y vertical vibrations, the resultant Lissajous figure will be constant in shape — the particular shape depending upon the phase difference of the two components. If x horizontal vibrations occur in almost exactly the same time as y vertical vibrations, then the phase difference will gradually change and the resultant Lissajous figure will go through the whole series of forms during the time one component gains one complete vibration over the other. This fact furnishes a precise method for the comparison of the periods of two vibrating bodies.

9. Fourier's Theorem. — Any periodic curve, harmonic or non-harmonic, which is continuous, having only one ordinate for each abscissa, and this ordinate of finite value, may be formed by an infinite number of combinations of harmonic curves. Fourier has shown that one of these combinations consists of harmonic curves whose wave-lengths are contained an exact number of times in the wave-length of the compound curve. This important theorem may be expressed in various ways, among which is the following:

Any periodic curve of wave-length λ having only single-valued ordinates of finite length may be produced by compounding harmonic curves (possibly infinite in number), having the same axis and wave-lengths λ, $\frac{1}{2}\lambda$, $\frac{1}{3}\lambda$, etc. The component curves of wave-lengths λ, $\frac{1}{2}\lambda$, $\frac{1}{3}\lambda$, etc., are called the *harmonics* of the resultant curve. For example, in Fig. 5 is represented a periodic curve, no ordinate of which cuts the curve more than once, and no ordinate of which is infinite in value. Fourier's theorem states

that among the sets of harmonic curves whose ordinates add up to that of the given curve, there is one set (and only one) which consists of harmonic curves whose wave-lengths are contained an exact number of times in the wave-length of the given curve.

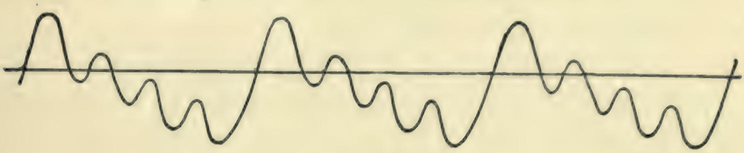

FIG. 5.

The equation of the resultant of any number of harmonic curves may be written

$$y = a_0 + \begin{cases} a_1 \sin\theta + a_2 \sin 2\,\theta + a_3 \sin 3\,\theta + \cdots \\ b_1 \cos\theta + b_2 \cos 2\,\theta + b_3 \cos 3\,\theta + \cdots \end{cases} \tag{17}$$

The term a_0 is the constant distance between the axis of abscissæ and the axis of the curve. Each sine term and each cosine term represents a component simple harmonic curve. Each a and each b represents the amplitude of the corresponding component. The component curves of the sine series, and also those of the cosine series have frequencies 1, 2, 3, etc. Hence the wave-lengths are 1, $\frac{1}{2}$, $\frac{1}{3}$, etc.

By a change of notation this equation may be put into a sine form. Thus, putting

$$M_1 = \sqrt{a_1^2 + b_1^2}, \quad M_2 = \sqrt{a_2^2 + b_2^2}, \text{ etc.,}$$

and
$$e_1 = \tan^{-1}\frac{b_1}{a_1}, \quad e_2 = \tan^{-1}\frac{b_2}{a_2}, \text{ etc.,}$$

$$y = a_0 + M_1 \sin(\theta + e_1) + M_2 \sin(2\,\theta + e_2) + \cdots. \tag{18}$$

θ is called the phase angle, and e is called the epoch.

Representing the period by T, the phase angle is often represented by $\theta = \frac{2\,\pi t}{T} \equiv kt$, so that (18) may be written

$$y = a_0 + M_1 \sin(kt + e_1) + M_2 \sin(2\,kt + e_2) + \cdots. \tag{19}$$

By giving proper values to the constants, the curve represented by (17), (18), or (19) can be made to pass through any assigned points. Suppose that it be required that within one-half wave-length this curve coincide with the curve $y = f(\theta)$ in n points. If the coördinates of any particular point of the curve $y = f(\theta)$ satisfy the equation

$$y = a_1 \sin \theta \tag{20}$$

it follows that the first component of the compound periodic curve must pass through that point. Furthermore, if these coördinates be substituted in (20), a_1 can be determined.

In the same manner the resultant of the first two components

$$y = a_1 \sin \theta + a_2 \sin 2\,\theta$$

may be made to pass through two points of the curve $y = f(\theta)$, and the constant a_2 determined.

And, in general, the resultant of all the components may be made to pass through n points of $y = f(\theta)$, and all n constants determined.

For example, suppose that the curve

$$y = a_1 \sin \theta + a_2 \sin 2\,\theta + \ldots\; a_n \sin n\theta \tag{21}$$

is to pass through n points of the curve $y = f(\theta)$ so chosen that their projections on the axis of abscissæ are separated by equal spaces each equal to

$$\Delta\theta = \frac{\pi}{n+1}.$$

Then the abscissæ of the various points of intersection are $\Delta\theta$, $2\,\Delta\theta$, $3\,\Delta\theta$, etc., and the respective ordinates are $f(\Delta\theta)$, $f(2\,\Delta\theta)$, $f(3\,\Delta\theta)$, etc.

On substituting these values in (21), we obtain

$$\begin{aligned}
f(\Delta\theta) &= a_1 \sin \Delta\theta + a_2 \sin 2\,\Delta\theta + \ldots\; a_n \sin n\,\Delta\theta.\\
f(2\,\Delta\theta) &= a_1 \sin 2\,\Delta\theta + a_2 \sin 4\,\Delta\theta + \ldots\; a_n \sin 2\,n\,\Delta\theta.\\
f(3\,\Delta\theta) &= a_1 \sin 3\,\Delta\theta + a_2 \sin 6\,\Delta\theta + \ldots\; a_n \sin 3\,n\,\Delta\theta.\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
f(n\,\Delta\theta) &= a_1 \sin n\,\Delta\theta + a_2 \sin 2\,n\,\Delta\theta + \ldots\; a_n \sin n^2\,\Delta\theta.
\end{aligned}$$

Thus we have n equations of the first degree to determine the n coefficients, a_1, a_2, a_3, etc.

10. To fix the ideas, consider a numerical example. Suppose that in the distance of one-half wave-length the curve (21) is to intersect the curve

$$y = \theta$$

in five points equidistant from one another. The values of the five coefficients a_1, a_2, a_3, a_4, and a_5 are now to be determined.

Now

$$\Delta\theta\left[= \frac{\pi}{n+1}\right] = \frac{\pi}{6}$$

and the projections of the intersections on the axis of abscissæ are

$$\frac{\pi}{6}, \quad \frac{2\pi}{6}, \quad \frac{3\pi}{6}, \quad \frac{4\pi}{6}, \quad \text{and} \quad \frac{5\pi}{6}.$$

And since $y = \theta$, the ordinates have the same values.

On substituting these values in the general equation (21),

$$\frac{\pi}{6} = a_1 \sin\frac{\pi}{6} + a_2 \sin\frac{2\pi}{6} + \ldots a_5 \sin\frac{5\pi}{6}. \tag{22}$$

$$\frac{2\pi}{6} = a_1 \sin\frac{2\pi}{6} + a_2 \sin\frac{4\pi}{6} + \ldots a_5 \sin\frac{10\pi}{6}. \tag{23}$$

$$\frac{3\pi}{6} = a_1 \sin\frac{3\pi}{6} + a_2 \sin\frac{6\pi}{6} + \ldots a_5 \sin\frac{15\pi}{6}. \tag{24}$$

$$\frac{4\pi}{6} = a_1 \sin\frac{4\pi}{6} + a_2 \sin\frac{8\pi}{6} + \ldots a_5 \sin\frac{20\pi}{6}. \tag{25}$$

$$\frac{5\pi}{6} = a_1 \sin\frac{5\pi}{6} + a_2 \sin\frac{10\pi}{6} + \ldots a_5 \sin\frac{25\pi}{6}. \tag{26}$$

The determination of the five coefficients in the above five equations is most easily accomplished by the following mathematical process due to Lagrange. First, multiply (22) by $2 \sin\frac{\pi}{6}$, (23) by $2 \sin\frac{2\pi}{6}$, (24) by $2 \sin\frac{3\pi}{6}$, and so on. Add the resulting equations, collecting all the factors of a_1 in one term, those of a_2 in another term, and so on. The coefficient a_1 can then be found.

As the process is long and somewhat involved it will now be considered in detail. After multiplying the above equations by the various factors and adding the resulting equations, we have:

$$2\,\frac{\pi}{6}\sin\frac{\pi}{6}+2\,\frac{2\pi}{6}\sin\frac{2\pi}{6}+2\,\frac{3\pi}{6}\sin\frac{3\pi}{6}+2\,\frac{4\pi}{6}\sin\frac{4\pi}{6}+2\,\frac{5\pi}{6}\sin\frac{5\pi}{6}$$

$$= a_1\left(2\sin^2\frac{\pi}{6}+2\sin^2\frac{2\pi}{6}+2\sin^2\frac{3\pi}{6}+2\sin^2\frac{4\pi}{6}+2\sin^2\frac{5\pi}{6}\right)$$

$$+a_2\left(2\sin\frac{\pi}{6}\sin\frac{2\pi}{6}+2\sin\frac{2\pi}{6}\sin\frac{4\pi}{6}+2\sin\frac{3\pi}{6}\sin\frac{6\pi}{6}+2\sin\frac{4\pi}{6}\sin\frac{8\pi}{6}+2\sin\frac{5\pi}{6}\sin\frac{10\pi}{6}\right)$$

$$+a_3\left(2\sin\frac{\pi}{6}\sin\frac{3\pi}{6}+2\sin\frac{2\pi}{6}\sin\frac{6\pi}{6}+2\sin\frac{3\pi}{6}\sin\frac{9\pi}{6}+2\sin\frac{4\pi}{6}\sin\frac{12\pi}{6}+2\sin\frac{5\pi}{6}\sin\frac{15\pi}{6}\right)$$

$$+a_4\left(2\sin\frac{\pi}{6}\sin\frac{4\pi}{6}+2\sin\frac{2\pi}{6}\sin\frac{8\pi}{6}+2\sin\frac{3\pi}{6}\sin\frac{12\pi}{6}+2\sin\frac{4\pi}{6}\sin\frac{16\pi}{6}+2\sin\frac{5\pi}{6}\sin\frac{20\pi}{6}\right)$$

$$+a_5\left(2\sin\frac{\pi}{6}\sin\frac{5\pi}{6}+2\sin\frac{2\pi}{6}\sin\frac{10\pi}{6}+2\sin\frac{3\pi}{6}\sin\frac{15\pi}{6}+2\sin\frac{4\pi}{6}\sin\frac{20\pi}{6}+2\sin\frac{5\pi}{6}\sin\frac{25\pi}{6}\right) \quad (27)$$

We will now determine the value of the coefficient of a_1. Since $2\sin^2 x = 1 - \cos 2x$, we have for the coefficient of a_1,

$$2\sin^2\frac{\pi}{6}+2\sin^2\frac{2\pi}{6}+2\sin^2\frac{3\pi}{6}+2\sin^2\frac{4\pi}{6}+2\sin^2\frac{5\pi}{6}$$
$$=5-\left(\cos\frac{2\pi}{6}+\cos\frac{4\pi}{6}+\cos\frac{6\pi}{6}+\cos\frac{8\pi}{6}+\cos\frac{10\pi}{6}\right).$$

The trigonometric series within the parenthesis is of a standard form which can be summed by means of the formula, proved in the Appendix,

$$\cos x+\cos 2x+\cos 3x+\cdots=-\frac{1}{2}+\frac{1}{2}\,\frac{\sin\,(2n+1)\dfrac{x}{2}}{\sin\dfrac{x}{2}}. \quad (28)$$

In the present case $n = 5$ and $x = \dfrac{2\pi}{6}$. Whence,

$$\text{Coeff. of } a_1 = 5+\tfrac{1}{2}-\frac{\sin\dfrac{11\pi}{12}}{2\sin\dfrac{\pi}{6}} = 5\tfrac{1}{2}-\frac{\sin\left(2\pi-\dfrac{\pi}{6}\right)}{2\sin\dfrac{\pi}{6}} = 6. \quad (29)$$

We will now determine the value of the coefficient of a_2. Since

$$2\sin x\sin 2x = \cos x - \cos 3x,$$

the first term of the coefficient of a_2 (27),

$$2\sin\frac{\pi}{6}\sin\frac{2\pi}{6} = \cos\frac{\pi}{6} - \cos\frac{3\pi}{6}.$$

Making a similar substitution in the other terms, we see that the coefficient of a_2 (27) can be put into the form,

$$\left(\cos\frac{\pi}{6} + \cos\frac{2\pi}{6} + \cos\frac{3\pi}{6} + \cos\frac{4\pi}{6} + \cos\frac{5\pi}{6}\right)$$
$$- \left(\cos\frac{3\pi}{6} + \cos\frac{6\pi}{6} + \cos\frac{9\pi}{6} + \cos\frac{12\pi}{6} + \cos\frac{15\pi}{6}\right).$$

Summing these two series by means of (28), we have

$$\text{Coefficient of } a_2 = -\frac{1}{2} + \frac{1}{2}\frac{\sin\frac{11}{12}\pi}{\sin\frac{\pi}{12}} - \left(-\frac{1}{2} + \frac{1}{2}\frac{\sin\frac{33}{12}\pi}{\sin\frac{3}{12}}\right).$$

But $\quad \frac{11}{12}\pi = \pi - \frac{1}{12}\pi \quad$ and $\quad \frac{33}{12}\pi = 3\pi - \frac{3}{12}\pi.$

Whence,

$$\text{Coefficient of } a_2 = \frac{\sin \pi - \frac{\pi}{12}}{2\sin\frac{\pi}{12}} - \frac{\sin\left(3\pi - \frac{3\pi}{12}\right)}{2\sin\frac{3\pi}{12}} = \frac{1}{2} - \frac{1}{2} = 0.$$

Consequently, the term of (27) containing a_2 vanishes. In the same manner it may be shown that the terms of (27) containing a_3, a_4, and a_5 also vanish.

Equation (27) may now be put into the form

$$2\left(\frac{\pi}{6}\sin\frac{\pi}{6} + \frac{2\pi}{6}\sin\frac{2\pi}{6} + \frac{3\pi}{6}\sin\frac{3\pi}{6} + \frac{4\pi}{6}\sin\frac{4\pi}{6} + \frac{5\pi}{6}\sin\frac{5\pi}{6}\right)$$
$$= 6\,a_1 + 0 + 0 + 0 + 0,$$

or, $$a_1 = \frac{2}{6}\sum_{k=1}^{5}\frac{k\pi}{6}\sin\frac{k\pi}{6}. \qquad (30)$$

Whence,

$$a_1 = \frac{2\pi}{6\times 6}\left(\frac{1}{2} + \sqrt{3} + 3 + 2\sqrt{3} + \frac{5}{2}\right) = \frac{\pi}{18}(6 + 3\sqrt{3}) \doteq 2.$$

To find a_2, multiply (22) by $2\sin\frac{2\pi}{6}$, (23) by $2\sin\frac{4\pi}{6}$, (24) by $2\sin\frac{6\pi}{6}$, (25) by $2\sin\frac{8\pi}{6}$, (26) by $\frac{10\pi}{6}$, and proceed exactly as in finding a_1. It will then be found that

$$a_2 = \frac{2}{6}\sum_{k=1}^{5}\frac{k\pi}{6}\sin\frac{2k\pi}{6} = -\frac{\pi}{6}\sqrt{3} = -0.9. \tag{31}$$

Similarly,

$$a_3 = \frac{2}{6}\sum_{k=1}^{5}\frac{k\pi}{6}\sin\frac{3k\pi}{6} = 0.5, \tag{32}$$

$$a_4 = \frac{2}{6}\sum_{k=1}^{5}\frac{k\pi}{6}\sin\frac{4k\pi}{6} = -0.3, \tag{33}$$

$$a_5 = \frac{2}{6}\sum_{k=1}^{5}\frac{k\pi}{6}\sin\frac{5k\pi}{6} = 0.1. \tag{34}$$

Substituting these coefficients in (27), the equation of the curve of the general form (21), which, within a distance of one-half wave-length, will intersect the curve $y = \theta$ at five points having as abscissæ $\frac{\pi}{6}$, $\frac{2\pi}{6}$, $\frac{3\pi}{6}$, $\frac{4\pi}{6}$, and $\frac{5\pi}{6}$, is seen to be

$$y = 2\sin\theta - 0.9\sin 2\theta + 0.5\sin 3\theta - 0.3\sin 4\theta + 0.1\sin 5\theta. \tag{35}$$

11. Damped Angular Vibration. — The angular motion of a body which at any instant is urged toward a position of equilibrium by a torque proportional to its angular displacement from that position is called simple harmonic motion of rotation. If in addition to this torque the body is acted upon by another torque about the same axis, due to friction or any retarding force, the resulting motion is called a *damped vibration*. Experiment shows that if the angular speed of any rotating system is not too great, the retarding or damping torque will be nearly proportional to the angular speed of the moving system.

To fix the ideas, consider a body suspended by a thin vertical wire and immersed in a viscous liquid. Let the suspended body be set into angular vibration about a line passing through the axis of the suspending wire. For an angular displacement of ϕ radians

from the position of equilibrium it has been shown (2) that the torque due to the elasticity of the wire is

$$L_1 = -\beta\phi.$$

If the angular speed of the body at this instant be represented by w, then since the damping torque for small angular displacements is nearly proportional to the angular speed, we may write for the damping torque,

$$L_2 = \alpha w,$$

where α is the so-called "damping factor" depending upon the viscosity of the liquid surrounding the moving body, or upon whatever other cause may produce damping.

Whence, the resultant torque

$$L\,[= L_1 - L_2] = -\beta\phi - \alpha w.$$

If the rotating system has a moment of inertia K, then the angular acceleration is

$$\frac{d^2\phi}{dt^2}\left[=\frac{L}{K}\right] = -\frac{\beta\phi}{K} - \frac{\alpha}{K}\frac{d\phi}{dt},$$

or,

$$K\frac{d^2\phi}{dt^2} + \alpha\frac{d\phi}{dt} + \beta\phi = 0. \qquad (36)$$

This equation is now to be solved for ϕ. It being a linear equation with constant coefficients, we will proceed to solve it by putting $\phi = \epsilon^{bt}$, where b is a constant whose value must be determined, and ϵ is the base of the natural logarithms.

Since

$$\phi = \epsilon^{bt}, \quad \frac{d\phi}{dt} = b\epsilon^{bt}, \quad \text{and} \quad \frac{d^2\phi}{dt^2} = b^2\epsilon^{bt},$$

the above equation may be written

$$\epsilon^{bt}\,(Kb^2 + \alpha b + \beta) = 0. \qquad (37)$$

Now ϵ^{bt} does not vanish for any finite value of b or t. Whence, in order that this equation may be satisfied,

$$Kb^2 + \alpha b + \beta = 0. \qquad (38)$$

The solution of this quadratic equation is

$$b = \frac{-\alpha \pm \sqrt{\alpha^2 - 4K\beta}}{2K} = -\frac{\alpha}{2K} \pm \sqrt{\frac{\alpha^2}{4K^2} - \frac{\beta}{K}}. \qquad (39)$$

An inspection of this equation shows that two cases can be distinguished — first, when $\frac{\alpha^2}{4K^2} \geqq \frac{\beta}{K}$; and second, when $\frac{\alpha^2}{4K^2} < \frac{\beta}{K}$. In the first case the roots of the equation are real, while in the second case the roots are imaginary.

First Case: Motion Non-oscillatory.

When, in (39), $\frac{\alpha^2}{4K^2} >$ or $= \frac{\beta}{K}$, the roots of the equation are real and are given by the expression

$$b = -\frac{\alpha}{2K} \pm j_1, \qquad (40)$$

where

$$j_1 = \left(\frac{\alpha^2}{4K^2} - \frac{\beta}{K}\right)^{\frac{1}{2}}. \qquad (41)$$

Denoting these two real roots by b' and b'',

$$b' = -\frac{\alpha}{2K} - j_1 \quad \text{and} \quad b'' = -\frac{\alpha}{2K} + j_1.$$

Since b' and b'' satisfy (38), then (37) shows that $\phi_1 = \epsilon^{b't}$ and $\phi_2 = \epsilon^{b''t}$ are solutions of (36). And if b' and b'' are distinct roots of (40), the solution of (36) will be of the form

$$\phi = C'\epsilon^{b't} + C''\epsilon^{b''t}.$$

Substituting the values of b' and b'' in this equation,

$$\begin{aligned} \phi &= C'\epsilon^{-\frac{\alpha t}{2K}}\epsilon^{-j_1t} + C''\epsilon^{-\frac{\alpha t}{2K}}\epsilon^{j_1t} \\ &= \epsilon^{-\frac{\alpha t}{2K}}(C'\epsilon^{-j_1t} + C''\epsilon^{j_1t}). \end{aligned} \qquad (42)$$

The constants of integration C' and C'' are now to be determined. Since this equation is true whatever the time, it is true when $t = 0$. At zero time, $\phi = 0$. Consequently, at this time,

$$0 = 1\,(C' + C'').$$

Therefore,

$$C' = -C''.$$

Again, if the angular speed at zero time be represented by w_0, we shall have on differentiating (42),

$$\frac{d\phi}{dt} = -\frac{\alpha}{2K}(C' + C'') + (-j_1C' + j_1C'') = w_0.$$

But since $C' = -C''$,

$$\frac{d\phi}{dt} = 0 - 2j_1C' = w_0.$$

Whence, $C' = -\frac{w_0}{2j_1}$ and $C'' = \frac{w_0}{2j_1}$.

Finally, substituting the values of these constants in (42), we obtain for the value of the angular displacement at any time t,

$$\phi = \frac{w_0}{2j_1}\epsilon^{-\frac{\alpha t}{2K}}(\epsilon^{j_1t} - \epsilon^{-j_1t}). \tag{43}$$

On examining this equation together with the equation obtained by differentiating it, it is found that when $t = 0$, then $\phi = 0$; that as t increases ϕ at first increases, but that after t exceeds a certain maximum value ϕ diminishes; and that ϕ continues to diminish until, when $t = \infty$, ϕ again becomes equal to zero.

Second Case: Motion Oscillatory.

When $\frac{\alpha^2}{4K^2} < \frac{\beta}{K}$, the roots of (39) are imaginary and are given by the expression

$$b = -\frac{\alpha}{2K} \pm j_2i, \tag{44}$$

where $j_2 = \left(-\frac{\alpha^2}{4K^2} + \frac{\beta}{K}\right)^{\frac{1}{2}}$ and $i = \sqrt{-1}$. (45)

Denoting these two roots by b_1 and b_2, we have

$$b_1 = -\frac{\alpha}{2K} + j_2i \quad \text{and} \quad b_2 = -\frac{\alpha}{2K} - j_2i.$$

If b_1 and b_2 satisfy (44), then (37) shows that $\phi = \epsilon^{bt}$ is a solution of (36). And if b_1 and b_2 are distinct roots of (44), the solution of (36) will be of the form

$$\phi = C_1\epsilon^{b_1t} + C_2\epsilon^{b_2t}.$$

Substituting the values of b_1 and b_2 in this equation,

$$\phi = C_1 \epsilon^{-\frac{\alpha t}{2K}} \epsilon^{j_2 i t} + C_2 \epsilon^{-\frac{\alpha t}{2K}} \epsilon^{-j_2 i t}.$$

Since, from trigonometry,

$$\epsilon^{j_2 i t} = \cos j_2 t + i \sin j_2 t \quad \text{and} \quad \epsilon^{-j_2 i t} = \cos j_2 t - i \sin j_2 t,$$

we can write

$$\begin{aligned} \phi &= \epsilon^{-\frac{\alpha t}{2K}} [C_1 \cos j_2 t + C_1 i \sin j_2 t + C_2 \cos j_2 t - C_2 i \sin j_2 t] \\ &= \epsilon^{-\frac{\alpha t}{2K}} [A \cos j_2 t + \beta \sin j_2 t], \end{aligned} \tag{46}$$

where $A = C_1 + C_2$ and $B = C_1 - C_2$.

The values of the constants A and B will now be determined. Since (46) is true for any value of t, it will hold when $t = 0$. When $t = 0$, then $\phi = 0$, and hence $A = 0$. If the angular speed of the suspended system when passing through the zero position be denoted by w_0, we shall have on differentiating (46) and remembering that $A = 0$ and $t = 0$,

$$\frac{d\phi}{dt} = w_0 = Bj_2.$$

Whence, $$A = 0 \quad \text{and} \quad B = \frac{w_0}{j_2}.$$

On substituting these values in (46),

$$\phi = \frac{w_0}{j_2} \epsilon^{-\frac{\alpha t}{2K}} \sin j_2 t, \tag{47}$$

$$\frac{d\phi}{dt} = \frac{w_0}{j_2} \epsilon^{-\frac{\alpha t}{2K}} \left[j_2 \cos j_2 t - \frac{\alpha}{2K} \sin j_2 t \right]. \tag{48}$$

Equations (47) and (48) show that both the angular displacement and angular speed are periodic quantities. Consequently, when the condition holds under which these equations were derived, namely, that $\frac{\alpha^2}{4K^2}$ is less than $\frac{\beta}{K}$, the motion is oscillatory.

The period of the vibration will now be deduced. Equation

(48) shows that if the velocity is zero when $t = t_1$, it will also be zero when

$$t = t_1 + \frac{\pi}{j_2}; \quad t = t_1 + \frac{2\pi}{j_2}; \quad t = t_1 + \frac{3\pi}{j_2}; \quad \text{etc.}$$

Consequently, the period T' of the damped vibration is

$$T' = \frac{2\pi}{j_2}. \tag{49}$$

Since the quantity j_2 depends upon the damping factor α (45), and this is nearly constant only when the damping is small, it follows that a damped vibration has a constant period only when the damping is very small. When the damping is large, the period is not constant and hence the vibration cannot be called periodic. It is also inexact to call the undulation produced by an unperiodic disturbance which is handed on successively from one portion of a medium to another, a wave motion.

On substituting in (47) the value of j_2 from (49), we obtain for the value of the angular deflection from its equilibrium position, at any time t, of a system vibrating under the influence of a retarding force which varies directly with the angular velocity of the moving system,

$$\phi = \frac{w_0 T'}{2\pi} \epsilon^{-\frac{\alpha t}{2K}} \sin \frac{2\pi t}{T'}. \tag{50}$$

If the vibration of period T' had been undamped, we find from (13) that the amplitude of vibration would be

$$\Phi = \frac{w_0 T'}{2\pi}.$$

Whence,

$$\phi = \Phi \epsilon^{-\frac{\alpha t}{2K}} \sin \frac{2\pi t}{T'}, \tag{51}$$

where Φ is the amplitude the vibration would have if there were no damping.

12. The Logarithmic Decrement. — It will now be shown that in the case of damped vibrations, if a series of turning points be read, these readings can be used for calculating the amplitudes

that would have been produced if there had been no decrease of the amplitude by damping.

Imagine a series of oscillations represented by the zig-zag line *abcdefgh*, etc., where *ace*, etc., are the positions of equilibrium of the various oscillations. Let the amplitudes of the successive swings to the right, ab ($= bc$), ef ($= fg$), etc., be represented by the symbols ϕ_1, ϕ_2, etc., and let the amplitude of the successive swings to the left, cd ($= de$), gh ($= hi$), etc., be represented by the symbols ϕ_1', ϕ_2', etc. Let time be reckoned from the position a. Then the amplitude of the next swing, that is, the displacement in time $t = \frac{T'}{4}$, is (51)

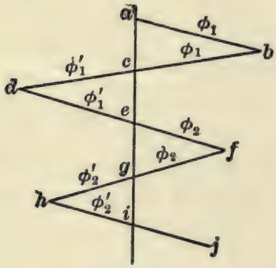

FIG. 6.

$$\phi_1 = \Phi\epsilon^{-\frac{\alpha T'}{8K}}.$$

Similarly, the amplitudes of the succeeding swings, that is the displacements at the time $t = \frac{3}{4}T'$, $\frac{5}{4}T'$, $\frac{7}{4}T'$, etc., are

$$\phi_1' = \Phi\epsilon^{-\frac{3\alpha T'}{8K}},$$

$$\phi_2 = \Phi\epsilon^{-\frac{5\alpha T'}{8K}}, \quad \text{etc.}$$

Whence, the oscillation

$$(bd)\ [= \phi_1 + \phi_1'] = \Phi\epsilon^{-\frac{\alpha T'}{8K}}\left(1 + \epsilon^{-\frac{\alpha T'}{4K}}\right),$$

$$(df)\ [= \phi_1' + \phi_2] = \Phi\epsilon^{-\frac{3\alpha T'}{8K}}\left(1 + \epsilon^{-\frac{\alpha T'}{4K}}\right), \quad \text{etc.}$$

Consequently,

$$\frac{bd}{df} = \epsilon^{\frac{\alpha T'}{4K}}.$$

Proceeding in the same manner,

$$\frac{bd}{df} = \frac{df}{fh} = \frac{fh}{hj} = \text{etc.} = \epsilon^{\frac{\alpha T'}{4K}} \equiv \sigma. \tag{52}$$

For all ordinary speeds, the damping factor α is so nearly constant that the quantity σ is practically constant. That is, in so far as an experiment will show, the ratio of any oscillation to the

next succeeding one is a constant quantity. The natural logarithm of this ratio is called the *logarithmic decrement* of the vibration and is usually represented by the symbol λ. Thus, the logarithmic decrement of the vibration is

$$\lambda = \frac{\alpha T'}{4K}. \tag{53}$$

Now if the motion had been undamped, the displacement at any time t would have had the magnitude (11)

$$\phi_u = \Phi \sin \frac{2\pi t}{T}.$$

Whereas, if the vibration were damped and the period of the damped vibration were the same as that of the undamped vibration, the corresponding displacement would have been (51)

$$\phi_d = \Phi \epsilon^{-\frac{\alpha t}{2K}} \sin \frac{2\pi t}{T}.$$

So that

$$\frac{\phi_u}{\phi_d} = \epsilon^{\frac{\alpha t}{2K}}. \tag{54}$$

Or, at the end of the swing, that is, when $t = \frac{T}{4}$,

$$\frac{\Phi_u}{\Phi_d} = \epsilon^{\frac{\alpha T}{8K}}, \tag{55}$$

where Φ_u and Φ_d represent the amplitudes of the undamped and the corresponding damped vibration, respectively. By means of (53) the above equation becomes

$$\frac{\Phi_u}{\Phi_d} = \epsilon^{\frac{\lambda}{2}}. \tag{56}$$

Or,

$$\Phi_u = \Phi_d \epsilon^{\frac{\lambda}{2}}.$$

On expanding $\epsilon^{\frac{\lambda}{2}}$,

$$\Phi_u = \Phi_d\left(1 + \frac{\lambda}{2} + \frac{\lambda^2}{8} + \frac{\lambda^3}{48} \cdots\right) \tag{57}$$

$$\doteqdot \Phi_d\left(1 + \frac{\lambda}{2}\right), \tag{58}$$

if λ is so small that $\frac{\lambda^2}{8}$ is negligible.

In finding the values of Φ_u by means of the logarithmic decrement, the observation of a number of turning points is required. For many purposes, however, it is sufficient to employ a method of determining Φ_u which requires the observation of but three consecutive turning points. Thus, denoting the magnitude of the consecutive oscillations, *i.e.*, the distances between consecutive turning points, by ξ_1, ξ_2, etc., we have from (52),

$$\frac{\xi_1}{\xi_2} = \epsilon^{\frac{\alpha T'}{4K}},$$

where T' is the period of the damped vibration.

If the period of the damped vibration be the same as the period of the undamped vibration from which we may consider it to be derived, we have from (55),

$$\Phi_u = \Phi_d \epsilon^{\frac{\alpha T'}{8K}}.$$

On combining this with the preceding equation,

$$\Phi_u = \Phi_d \left(\frac{\xi_1}{\xi_2}\right)^{\frac{1}{2}}. \tag{59}$$

13. Computation of the Logarithmic Decrement. — In the preceding article it has been shown that within the limits of ordinary experiment, the ratio of any oscillation to the next succeeding one is a constant quantity. Thus, if a series of successive oscillations be represented by ξ_1, ξ_2, etc., we shall have

$$\frac{\xi_1}{\xi_2} = \frac{\xi_2}{\xi_3} = \frac{\xi_3}{\xi_4} = \cdots = \epsilon^{\lambda},$$

where ϵ is the base of the Naperian logarithms and λ is the logarithmic decrement.

Then

$$\left(\frac{\xi_1}{\xi_2}\right) = \left(\frac{\xi_1}{\xi_3}\right)^{\frac{1}{2}} = \left(\frac{\xi_1}{\xi_4}\right)^{\frac{1}{3}} = \cdots = \left(\frac{\xi_1}{\xi_m}\right)^{\frac{1}{m-1}} = \cdots = \epsilon^{\lambda}.$$

When the damping is very small, the ratio of two succeeding oscillations is nearly equal to unity. In this case it is preferable to

employ the ratio of two oscillations that are not consecutive. Let it be required to obtain the ratio of the mth to the nth oscillation.

$$\frac{\xi_m}{\xi_n} = \frac{\dfrac{\xi_1}{\xi_n}}{\dfrac{\xi_1}{\xi_m}} = \frac{(\epsilon^\lambda)^{n-1}}{(\epsilon^\lambda)^{m-1}} = (\epsilon^\lambda)^{n-m},$$

or,

$$\left(\frac{\xi_m}{\xi_n}\right)^{\frac{1}{n-m}} = \epsilon^\lambda.$$

Consequently,

$$\frac{1}{n-m}\left(\log_e \frac{\xi_m}{\xi_n}\right) = \frac{1}{(n-m)\,(0.4343)} \log_{10} \frac{\xi_m}{\xi_n} = \lambda. \qquad (60)$$

In computing oscillations it should be remarked that if the zero of the scale lies between two consecutive turning points, the oscillations will be given by the sums of the successive readings; if, however, the zero of the scale is not between two successive turning points, the oscillations will be given by the difference between the successive readings. The method of tabulating observations and derived results is illustrated in the following example.

DETERMINATION OF THE LOGARITHMIC DECREMENT OF A WIEDEMANN GALVANOMETER ARRANGED BALLISTICALLY

ZERO POINT AT MIDDLE OF THE SCALE

Turning points		Oscillations
Left	Right	
	5.02	
4.98		1st — 5.02 + 4.98 = 10.00
	4.84	2d — 4.98 + 4.84 = 9.82
4.79		3d — 4.84 + 4.79 = 9.63
	4.62	4th — 4.79 + 4.62 = 9.41
4.60		5th — 4.62 + 4.60 = 9.22
	4.48	6th — 4.60 + 4.48 = 9.08
4.44		7th — 4.48 + 4.44 = 8.92
	4.32	8th — 4.44 + 4.32 = 8.76
4.30		9th — 4.32 + 4.30 = 8.62
	4.18	10th — 4.30 + 4.18 = 8.48
4.10		11th — 4.18 + 4.10 = 8.28
	4.04	12th — 4.10 + 4.04 = 8.14
3.95		13th — 4.04 + 3.95 = 7.99
	3.90	14th — 3.95 + 3.90 = 7.85
3.81		15th — 3.90 + 3.81 = 7.71
	3.86	16th — 3.81 + 3.86 = 7.67

$$_{11}\lambda_1 = \frac{1}{(11-1)(0.4343)}\left(\log\frac{10.00}{8.28}\right) = 0.019$$

$$_{13}\lambda_3 = \frac{1}{(13-3)(0.4343)}\left(\log\frac{9.63}{7.99}\right) = 0.019$$

$$_{15}\lambda_5 = \frac{1}{(15-5)(0.4343)}\left(\log\frac{9.22}{7.71}\right) = 0.018.$$

etc.

The mean of the above values is the logarithmic decrement required.

14. Relation between the Period of a Damped and the Period of the Corresponding Undamped Vibration. — In the case of a simple harmonic motion of rotation, the period is (10)

$$T = 2\pi\sqrt{\frac{K}{\beta}},$$

while the period of the corresponding damped vibration is (49)

$$T' = \frac{2\pi}{j_2}.$$

Whence,

$$\frac{1}{T^2} - \frac{1}{T'^2} = \frac{1}{4\pi^2}\left(\frac{\beta}{K} - j_2^2\right).$$

Substituting for j_2 its value from (45),

$$\frac{1}{T^2} - \frac{1}{T'^2} = \frac{\alpha^2}{16\pi^2K^2}. \tag{61}$$

Substituting for K its value from (53),

$$\frac{1}{T^2} - \frac{1}{T'^2} = \frac{\lambda^2}{\pi^2 T'^2}.$$

Whence,

$$\frac{T'^2}{T^2} = 1 + \frac{\lambda^2}{\pi^2}. \tag{62}$$

Exp. 51. Graphical Construction of Lissajous Figures

Theory of the Experiment. — Read Arts. 1 and 8. The object of this experiment is to construct Lissajous figures corresponding to various phase differences of two simple harmonic vibrations of assigned periods and amplitudes.

Manipulation. — Proceeding as described in Art. 8, and illustrated in Fig. 2, divide each of the two construction semicircles into equal arcs. The ratio of the number of arcs into which the two semicircles is divided should equal the ratio of the periods of vibration of the component vibrations. For example, if the ratio between the periods of the vertical and the horizontal components be 3 : 5, then the vertical semicircle might be divided into six equal arcs and the horizontal component into ten equal arcs.

Construct curves for five phase differences.

In the drawing represent no amplitude of vibration by a line less than three inches in length. Draw all guide lines with a hard sharp pencil and draw the Lissajous figures with instruments in India ink.

Exp. 52. Determination of the Equation of a Compound Wave Form by Fourier's Harmonic Analysis

THEORY OF THE EXPERIMENT. — Read Arts. 1, 6, 7, 9, and 10. The object of this experiment is to plot the periodic curve represented by an assigned equation, and then, by the application of Fourier's method to this curve, to recover the given equation.

Suppose the distance of one-half wave-length along the axis of abscissæ of a periodic curve of the form (21) to be divided by n points into equal spaces each of length Δx. Then if $y = f(x)$, the ordinate of a point of the curve of abscissa $k\Delta x$ is $f(k\Delta x)$, and the phase angle is $(mk\,\Delta x)$. A comparison of (30), (31), (32), (33), and (34) shows that the value of any coefficient of (21) is

$$a_m = \frac{2}{n+1}\sum_{k=0}^{n} f(k\,\Delta x)\sin(mk\,\Delta x). \tag{63}$$

MANIPULATION. — The procedure will be illustrated by a concrete example. Suppose we have given the equation

$$y = 22 + 10\sin kt + 8\sin(2\,kt + 30) + 5\sin(3\,kt + 60). \tag{64}$$

It will be observed that this is the resultant of three sine curves, the first component curve having an amplitude of 10; the second an amplitude of 8, a wave-length one-half that of the first, and reaching its maximum displacement 30° before the first; and a third component having an amplitude of 5, a wave-length one-third that of the first, and leading the first by 60°.

The curve represented by (64) was plotted as follows: On a convenient axis, circles were drawn having radii proportional to the amplitudes of the three components, Fig. 7. On the same axis was laid off a suitable length representing one wave-length, and this distance was divided into 24 equal parts. The circumference

of the circles was divided into the same number of equal arcs, *i.e.*, into arcs of 15°.

To construct the first component, or fundamental, ordinates were erected from each of the 24 points on the axis of abscissæ. From the intersection of the auxiliary circle of radius 10 with the 24 radii, lines were drawn parallel to the axis of abscissæ. The

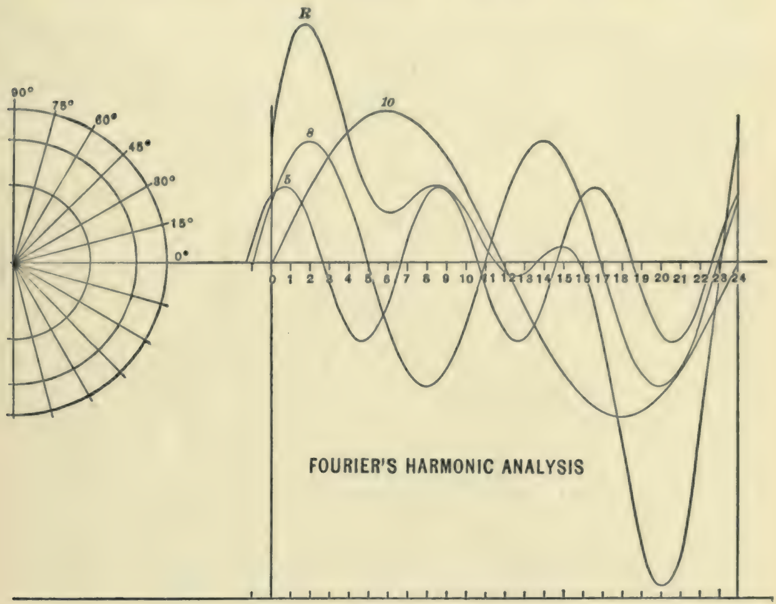

FIG. 7.

curve drawn through the intersections of the ordinates and the horizontal lines drawn from the corresponding points of the auxiliary circle is the fundamental curve.

Since the component of amplitude 8 leads the fundamental by 30°, it crosses the axis of ordinates at the instant the fundamental is at the origin, and at the point as far from the axis of abscissæ as is the intersection of the auxiliary circle of radius 8 with the 30° radius. Succeeding points of this component are obtained as for the fundamental, remembering, however, that the wave-length

corresponds to 12 divisions on the axis of abscissæ. This component is marked "8" in the figure.

In the same manner was drawn the component of amplitude 5 having a wave-length corresponding to 8 divisions on the axis of abscissæ and leading the fundamental by 60°.

The resultant of these three components, that is, the curve represented by the assigned equation (64), is found by adding the ordinates of the three components and drawing a curve through the end points, *R*, Fig. 7. The constant 22 which appears in the assigned equation signifies that the axis of the curve is 22 cm. from the axis of reference. In the figure the axis of reference is so indicated.

TABLE OF

I	II	III	IV	V	VI	VII	VIII
Time	y	kt	$\sin kt$	$y \sin kt$	$\cos kt$	$y \cos kt$	$\sin 2kt$
0	30.27	0	0	0	1.0000	30.2700	0
1	36.28	15	0.2588	9.3892	0.9659	35.0429	0.5000
2	37.72	30	0.5000	18.8600	0.8660	32.6655	0.8660
3	34.58	45	0.7071	24.4515	0.7071	24.4515	1.0000
4	30.22	60	0.8660	26.1705	0.5000	15.1100	0.8660
5	26.79	75	0.9659	26.1665	0.2588	6.9333	0.5000
6	25.35	90	1.0000	25.3500	0	0	0
7	25.84	105	0.9659	24.9588	−0.2588	− 6.6874	−0.5000
8	26.90	120	0.8660	23.2954	−0.5000	−13.4500	−0.8660
9	26.85	135	0.7071	18.9856	−0.7071	−18.9856	−1.0000
10	25.46	150	0.5000	12.7300	−0.8660	−22.0484	−0.8660
11	23.13	165	0.2588	5.9860	−0.9659	−22.3413	−0.5000
12	21.55	180	0	0	−1.0000	−21.5500	0
13	21.30	195	−0.2588	− 5.5124	−0.9659	−20.5737	0.5000
14	22.32	210	−0.5000	−11.1600	−0.8660	−19.3291	0.8660
15	22.97	225	−0.7071	−16.2420	−0.7071	−16.2421	1.0000
16	21.58	240	−0.8660	−18.6882	−0.5000	−10.7900	0.8660
17	17.14	255	−0.9659	−16.5555	−0.2588	− 4.5358	0.5000
18	10.41	270	−1.0000	−10.4100	0	0	0
19	3.90	285	−0.9659	− 3.7670	0.2588	1.0193	−0.5000
20	0.83	300	−0.8660	− 0.7187	0.5000	0.4150	−0.8660
21	3.00	315	−0.7071	− 2.1213	0.7071	2.1210	−1.0000
22	10.35	330	−0.5000	− 5.1750	0.8660	8.9631	−0.8660
23	20.57	345	−0.2588	− 5.3235	0.9659	19.8684	−0.5000
Mean:	21.88			5.03		0.01	

We will now find the equation of the resultant R, Fig. 7, and compare it with the assigned equation. From the reference line are measured the ordinates of the resultant curve. These values are placed in column II. The various phase angles are placed in column III, and their sines in column IV. In the other columns of the table are placed the data indicated by the headings. From these data can be computed the constants in (17) and (18).

Expressed in words, (63) states that any coefficient a_m is equal to twice the mean of the products of the ordinates and the sines of the corresponding phase angles.

DATA

IX	X	XI	XII	XIII	XIV	XV
$y \sin 2\,kt$	$\cos kt$	$y \cos 2\,kt$	$\sin 3\,kt$	$y \sin 3\,kt$	$\cos 3\,kt$	$y \cos 3\,kt$
0	1.0000	30.2700	0	0	1.0000	30.2700
18.1400	0.8660	31.4185	0.7071	25.6535	0.7071	25.6535
32.6655	0.5000	18.8600	1.0000	37.7200	0	0
34.5800	0	0	0.7071	24.4515	−0.7071	−24.4515
26.1705	−0.5000	−15.1100	0	0	−1.0000	−30.2200
13.3950	−0.8660	−23.2001	−0.7071	−18.9432	−0.7071	−18.9432
0	−1.0000	−25.3500	−1.0000	−25.3500	0	0
12.9200	−0.8660	−22.3774	−0.7071	−18.2714	0.7071	18.2714
−23.2954	−0.5000	−13.4500	0	0	1.0000	26.9000
−26.8500	0	0	0.7071	18.9856	0.7071	18.9856
−22.0483	0.5000	12.7300	1.0000	25.4600	0	0
−11.5650	0.8660	20.0306	0.7071	16.3552	−0.7071	−16.3552
0	1.0000	21.5500	0	0	−1.0000	−21.5500
10.6500	0.8660	18.4458	−0.7071	−15.0612	−0.7071	−15.0612
19.3291	0.5000	11.1600	−1.0000	−22.3200	0	0
22.9700	0	0	−0.7071	−16.2421	0.7071	16.2421
18.6882	−0.5000	−10.7900	0	0	1.0000	21.5800
8.5700	−0.8660	−14.8432	0.7071	12.1197	0.7071	12.1197
0	−1.0000	−10.4100	1.0000	10.4100	0	0
− 1.9500	−0.8660	− 3.3774	0.7071	2.7577	−0.7071	− 2.7577
− 0.7187	−0.5000	− 0.4150	0	0	−1.0000	− 0.8300
− 3.0000	0	0	−0.7071	− 2.1210	−0.7071	− 2.1210
− 8.9631	0.5000	5.1740	−1.0000	−10.3500	0	0
−10.2850	0.8660	17.8136	−0.7071	−14.5480	0.7071	14.5450
3.48		2.01		1.28		2.18

a_0 [= the mean of the values in column II] = 21.88
a_1 [= 2 (mean of the values in column V)] = 10.06
a_2 [= 2 (" " " " IX)] = 6.96
a_3 [= 2 (" " " " XIII)] = 2.56
b_1 [= 2 (" " " " VII)] = 0.03
b_2 [= 2 (" " " " XI)] = 4.02
b_3 [= 2 (" " " " XV)] = 4.36

$$M_1 \left[= \sqrt{a_1^2 + b_1^2}\right] = 10.04 \qquad \epsilon_1 \left[= \tan^{-1}\frac{b_1}{a_1}\right] = 0° \, 10',$$

$$M_2 \left[= \sqrt{a_2^2 + b_2^2}\right] = 8.06 \qquad \epsilon_2 \left[= \tan^{-1}\frac{b_2}{a_2}\right] = 30° \, 5',$$

$$M_3 \left[= \sqrt{a_3^2 + b_3^2}\right] = 5.02 \qquad \epsilon_3 \left[= \tan^{-1}\frac{b_3}{a_3}\right] = 59° \, 35'.$$

Substituting these values in (21), the equation of the resultant is found to be

$$y = 21.88 + 10.04 \sin (kt + 0° \, 10') + 8.06 \sin (kt + 30° \, 5') + 5.02 \sin (kt + 59° \, 35').$$

The departure of this equation from the assigned equation is due to errors and approximations in our work.

Exp. 53. Study of Damped Vibration

Theory of the Experiment. — Read Arts. (1–5) and (11–14). It has been shown that for a freely suspended body executing simple undamped harmonic motion of rotation, the angular displacement for any time t is (11)

$$\phi_u = \Phi_u \sin \frac{2\pi t}{T}, \tag{65}$$

where Φ is the amplitude and T is the period of the vibration.

If the motion be damped, the amplitude does not remain constant but becomes a function of time, and the angular displacement at any time t is (51)

$$\phi_d = \Phi_u \epsilon^{-\frac{at}{2K}} \sin \frac{2\pi t}{T'}, \tag{66}$$

or (53),

$$\phi_d = \Phi_u \epsilon^{-\frac{2\lambda t}{T'}} \sin \frac{2\pi t}{T'}, \tag{67}$$

where ϵ represents the Naperian base, λ the logarithmic decrement, T' the period of the damped vibration, and Φ_u is the amplitude the vibration would have if there were no damping.

In (58) it has been shown that the ratio between the amplitude of any undamped and the corresponding damped displacement is

$$\frac{\Phi_u}{\Phi_d} \doteq \left(1 + \frac{\lambda}{2}\right). \tag{68}$$

It is the object of the present experiment to determine the time-displacement curve of a suspended disk vibrating torsionally in a liquid, and, by means of (67) and (68), to construct the curve that would have resulted had there been no damping. The accuracy of the work will be checked by computing the magnitudes of several displacements by (67).

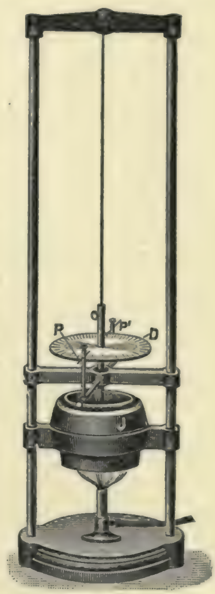

FIG. 8.

MANIPULATION. — The apparatus used in this experiment consists of a massive metal cylinder, immersed in oil and suspended axially by a steel wire from a rigid support. Fastened to the rod connecting the cylinder with the supporting wire is a divided circle, D, Fig. 8, and fastened to the rigid support are two pointers PP'.

Being careful not to give the cylinder any swinging motion, twist it through an angle of about 180° and release it so that it will oscillate torsionally. The length and diameter of the supporting wire must be such that the period shall be not less than 30 sec.

Angular deflections are now to be read at five-second intervals for twenty or more periods of vibration of the cylinder. These values plotted on cross-section paper, with time for abscissæ and angular displacements for ordinates, will give the damped vibra-

tion curve as shown in Fig. 9. (In this diagram it will be noted that minutes and seconds of time have been indicated by the symbols ′ and ″. This is not good practice, but was resorted to on account of the small space.)

The curve of undamped vibrations can be determined in the following manner. With a watch observe the time of ten complete

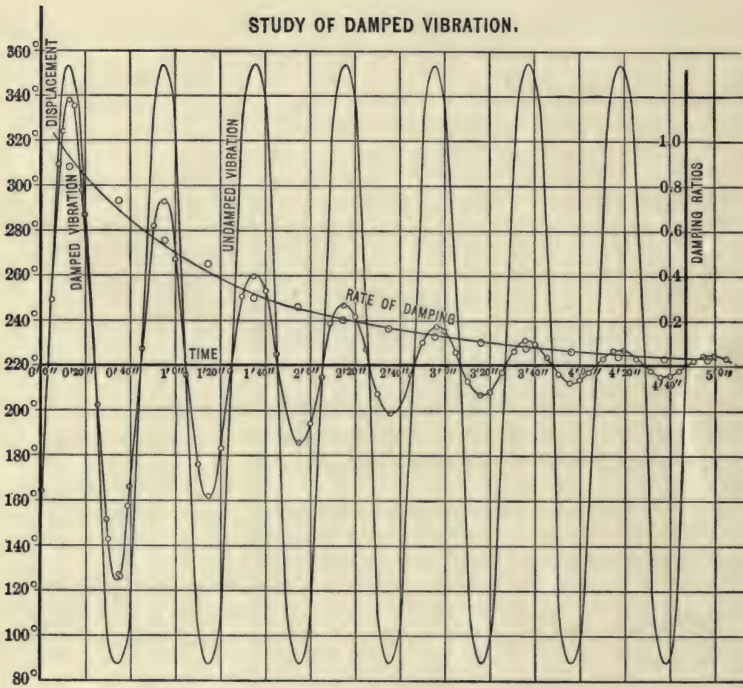

FIG. 9.

vibrations. Make a series of readings of the turning points of successive right and left oscillations and by means of (60) find the logarithmic decrement λ. From the damped vibration curve find the angular displacement at the first turning point, that is, the amplitude Φ_d for the time $t = \frac{T'}{4}$. By means of (68) determine the amplitude Φ_u of the corresponding undamped vibration. From

(62) the peirod T of the undamped vibration can be found. By substituting Φ_u and T in (65), and giving t any convenient values, as $\frac{T}{4}$, $\frac{T}{6}$, $\frac{T}{8}$, etc., the first cycle of the undamped curve can be plotted. As all the cycles of an undamped vibration are the same, other cycles can be laid off without further observation or computation.

By substituting in (67) Φ_u, T', and λ, and assigning to t a series of convenient values, a series of values for the angular displacements of the damped vibration may be computed. These values when plotted on the same axes should coincide with the experimentally observed damped vibration curve.

A curve showing the rate of damping of the amplitude of vibration of the cylinder may be constructed as follows: For a number of points on the axis of abscissæ separated by equal distances, find the ratio of the damped to the undamped displacement. At these points erect ordinates of lengths proportional to these ratios. The curve having these ordinates gives the rate of damping at any particular time.

Exp. 54. Determination of the Specific Viscosities of Liquids by the Vibrating Disk Method

Theory of the Experiment. — Read Arts. 11 and 12. By specific or relative viscosity is meant the ratio of the viscosity of a given liquid to the viscosity of water. It is shown in Art. 11 that if a massive disk suspended axially by a thin vertical wire be immersed in a liquid and set into torsional vibration, the ratio of the lengths of any two successive swings from one end of the path to the other is a known function of the "damping constant" α, which is proportional to the viscosity of the liquid surrounding the oscillating body. This is the basis of a method due to Coulomb for determining relative viscosities.

Let σ_1 represent the ratio of the lengths of any two successive oscillations of the disk when immersed in the first liquid; T_1', the period of vibration of the disk when immersed in the first liquid; and α_1, the damping constant for the first liquid. Let

σ_2, T_2', and α_2 represent the corresponding quantities for the second liquid. Then, from (52),

$$\sigma_1 = \epsilon^{\frac{\alpha_1 T_1'}{4K}} \tag{69}$$

and

$$\sigma_2 = \epsilon^{\frac{\alpha_2 T_2'}{4K}}, \tag{70}$$

where ϵ is the base of the natural logarithms and K is the moment of inertia of the suspended system. Dividing each member of (69) by the corresponding member of (70) and putting the resulting equation into the logarithmic form, we obtain

$$\frac{\log \sigma_1}{\log \sigma_2} = \frac{\alpha_1 T_1'}{\alpha_2 T_2'}.$$

Whence, the relative viscosity of the two liquids, z, is

$$z = \frac{\alpha_1}{\alpha_2} = \frac{T_2' \log \sigma_1}{T_1' \log \sigma_2}. \tag{71}$$

If the second liquid be water, z is the specific viscosity of the first liquid.

Manipulation. — In the apparatus here employed (Fig. 8), one end of a thin piano wire is fastened to a rigid support while the other end is attached to a vertical rod carrying a divided circle and the massive disk which is to be immersed in the various liquids. The disk has a thin stem by which it is fastened to the rod carrying the divided circle. The vessel containing the liquid being studied is surrounded by an oil bath heated by means of a Bunsen burner.

As the viscosity of many liquids is very different at different temperatures, it is always necessary to make the determination at the temperature at which the liquid is to be used. For instance, a test of cylinder oil should be made at about 150° to 175° C., while most machine oils should be tested at about 50° C. Since the relative viscosities of many pairs of specimens of oil are even reversed with a change of temperature of less than 100° C., it is impossible to judge the relative lubricating values of oils from their relative viscosities determined at a tempera-

ture much different from the temperature at which they are to be used.

After cleaning and assembling the apparatus and allowing the temperature of the specimen to attain the required value, twist the disk through about 180° by rotating the rod above the divided circle. With a stop watch observe the time of ten complete vibrations. One-tenth of this time in seconds is the period T_1'. By means of the pointers P and P' (Fig. 8), make a series of readings of the turning points of successive swings to the right and to the left. The number of scale divisions through which the disk turns in rotating from one end of its path to the other is the magnitude of that oscillation. Calling the magnitudes of these successive oscillations ξ_1, ξ_2, ξ_3, etc., we have

$$\frac{\xi_1}{\xi_2} = \frac{\xi_2}{\xi_3} = \frac{\xi_3}{\xi_4} = \cdots \equiv \sigma_1.$$

Whence,

$$\begin{aligned} \xi_3 &= \xi_4\sigma_1, \\ \xi_2 &= \xi_3\sigma_1 = \xi_4\sigma_1^2, \\ \xi_1 &= \xi_2\sigma_1 = \xi_4\sigma_1^3, \end{aligned}$$

and, in general,

$$\xi_n = \xi_m\sigma_1^{m-n}.$$

If, say, twenty oscillations were observed, we have then

$$\begin{aligned} \xi_1 &= \xi_{11}\sigma_1^{10}, \\ \xi_2 &= \xi_{12}\sigma_1^{10}, \\ \xi_3 &= \xi_{13}\sigma_1^{10}, \text{ etc.,} \end{aligned}$$

and by finding the average of $\frac{\xi_1}{\xi_{11}}, \frac{\xi_2}{\xi_{12}}, \frac{\xi_3}{\xi_{13}}, \ldots \frac{\xi_{10}}{\xi_{20}}$, and taking the tenth root of this average, σ_1 is found.

In the same manner find T_2' and σ_2. These values of T_1', T_2', σ_1, and σ_2 substituted in (71) will give the relative viscosity of the two liquids. With liquids having viscosities not very different, the value of T_1' will be so nearly equal to T_2' that their ratio may approximate unity; but it is never allowable to assume their ratio to be unity without experimental verification.

CHAPTER II

SOUND

15. Resonance. — The vibrations executed by a body which has been displaced from its position of equilibrium and then released are called *free vibrations.* The vibrations executed under the action of an external periodically varying force are called *forced vibrations.* If the natural free period of the vibrating body be the same as the period of the external force, the amplitude of vibration will be large. The vibration produced when a periodically varying force acts upon a body of nearly the same period of vibration is called *resonant forced vibration.* The phenomenon of the production of resonant forced vibration is called *sympathetic vibration* or *resonance.*

If the mass of the receiving body is large and the viscous opposition to motion is small, as in the case of a tuning fork, resonance occurs only when the period of the exciting force is almost exactly equal to the period of the receiving body. But if the mass of the receiving body is small and the viscous opposition to motion is large, as in the case of air vibrating in a narrow tube, resonance will occur when the period of the exciting force is appreciably different from the free vibration period of the receiving body.

The above statements apply to the ordinary case in which the periodic exciting force varies harmonically. It should be noted, however, that if instead of varying harmonically, the impulses of the external periodic force start and stop suddenly, they will also set into resonant vibration a body whose free period is any exact submultiple of the period of the external force.

16. Beats. — Suppose two sound waves of different frequencies travel the same path in the same direction. If the frequencies of the two waves be n_1 and n_2 vibrations per second, respectively, then during one second the two waves will be in the same phase

$(n_1 - n_2)$ times. That is, during one second of time, the intensity of the resultant sound will rise to a maximum and fall to a minimum $(n_1 - n_2)$ times. The maxima of loudness are called *beats.*

The difference between the frequencies of two notes of nearly the same pitch can be readily determined by counting the number of beats occurring per second. For example, if when tuning forks X and Y are sounding, there are four beats per second, the difference in pitch is four vibrations per second. If one knows that the fork X is lower in pitch then Y, and also that it makes 256 complete vibrations per second, then the fork Y must make 260 complete vibrations per second.

17. Stationary Undulations in Air. — A wave going from one medium to another in which the speed is different will, in general, be divided at the interface separating the two media, — part of the wave entering the second medium and the remainder being reflected. If the incidence be normal, the reflected wave and the incident wave will be in the same straight line. If the speed be less in the second medium than in the first, and if the distance between the source and the reflecting surface be any odd number of quarter wave-lengths, the resultant motion will be a stationary vibration with a node at the reflecting surface and an antinode at the source. If, however, the speed be greater in the second medium than in the first, the resultant motion will be a stationary vibration when the distance between the source and the reflecting surface is any even number of quarter wave-lengths. In the latter case, there will be an antinode at the source and also at the reflecting surface.

For example, if a sound of constant pitch be produced at the open end of a tube of uniform bore closed at the other end, there will be set up within the tube a state of stationary undulation when the length of the tube is $\frac{1}{4}\lambda$, $\frac{3}{4}\lambda$, $\frac{5}{4}\lambda$, etc., where λ represents the wave-length of the sound. If a stationary undulation exist in a long tube of uniform bore, there will be a series of nodes one half-wave apart. If the frequency, n, and the wave-length, λ, be known, the approximate speed in the tube can be computed from the relation

$$v = n\lambda. \tag{72}$$

It should be noted that the speed of sound in a tube depends upon the diameter of the tube and also, to a slight extent, upon the roughness of the bore, the thermal conductivity of the material composing the walls of the tube, and upon the frequency of the sound. Up to the present date, these quantities have not been coördinated in an equation which will give results agreeing with experiment.

From stationary undulations of known frequency in the open air, that is, without any containing tube, Hebb* found the speed of sound in air at 0° C. to be 331.29 meters per second.

18. Electrically Driven Tuning Forks. — When a note or a mechanical vibration of constant frequency is to be maintained for some time, an electrically driven tuning fork is usually employed. This consists of a tuning fork provided with an electromagnet and battery so arranged that the vibrating prongs will cause the current in the coil to be made and broken once during each complete vibration. When the current traverses the electromagnet, the prongs of the fork are pulled away from the equilibrium position. When the current ceases, the released prongs fly back through the equilibrium position.

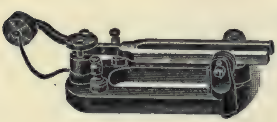

FIG. 10.

The make-and-break device most often employed is the same as that used on electric bells. See Figs. 15, 16, and 18.

A different device depends upon the great change of the contact resistance between pieces of carbon produced by small changes of pressure. In Fig. 10 is shown a capsule containing an arrangement similar to a telephone microphone attached to the base of a tuning fork provided with an electromagnet. The microphone and electromagnet are connected in series with a battery not shown in the figure. When the fork vibrates, the parts of the microphone will be alternately in close contact and in loose contact, thereby causing the current in the electromagnet to rise and fall with the frequency of vibration of the fork. This driving mechanism will operate forks of any frequency within the capacity of the frame to which the forks are mounted.

* Physical Review, p. 89, 1905.

19. The Velocity of a Longitudinal Wave in an Elastic Medium. — A particle having a position of equilibrium at a distance x from the origin of disturbance will, in time t, have moved from the position of equilibrium through a distance y. And a particle having a position of equilibrium $x + dx$ from the source will, in time t, be at a distance from the origin $x + dx + y + dy$.

Consider a lamina perpendicular to the line of motion which had an original thickness dx, and which now has a thickness $dx + dy$. Since the lamina has been distorted dy, it has undergone a strain $-(dy/dx)$.

Let P be the pressure on the lamina before the arrival of the disturbance, and p the pressure at any given instant after the disturbance reaches it. Then the change of pressure

$$\Delta P = p - P. \tag{73}$$

Representing the coefficient of elasticity of the medium by E,

$$\Delta P = -E\frac{dy}{dx}. \tag{74}$$

Now, since in time t a particle has moved through a distance y, the velocity u of the particle is $u = dy/dt$. And since, during this time, the disturbance has travelled a distance x, the velocity of the wave is $v = dx/dt$. Substituting these values of dy and dx in (74), and noting that ΔP and u must be of the same sign, we find that the variable increment of pressure on the element due to the wave is

$$\Delta P\left[= -E\frac{dy}{dx}\right] = E\frac{u\,dt}{v\,dt} = \frac{Eu}{v}. \tag{75}$$

The force which produces the acceleration of the element is dp. This is measured by the product of the mass and the acceleration of the element. Representing the density of the medium by ρ, the mass of an element of thickness dx and unit area of cross section is $\rho\,dx$. Hence the force which produces acceleration is

$$dp\left[= m\frac{du}{dt}\right] = \rho\,dx\frac{du}{dt}$$

$$= \rho\,dx\frac{du}{dt}\frac{dx}{dt} = \rho\,du \cdot v. \tag{76}$$

Remembering that P is a constant, we have from (73),

$$dp = d\,(\Delta P),$$

and remembering that E and v are constants, we have from (74),

$$d\,(\Delta P) = d\left(E\frac{u}{v}\right) = E\frac{du}{v}\cdot$$

Whence, $$dp = E\frac{du}{v}\cdot \tag{77}$$

From (76) and (77),

$$dp = \rho\, du \cdot v = E\frac{du}{v}\cdot$$

Whence, the velocity of the disturbance through the medium is

$$v = \sqrt{\frac{E}{\rho}}\cdot \tag{78}$$

The value of E is the coefficient of elasticity which applies to the sort of strain set up in the medium. For a solid rod or wire, free to expand transversely, the strain is tensile, and E is Young's modulus. For a fluid column, the strain is of volume and E is the bulk modulus.

The compressions and rarefactions of a sound wave are so rapid that the temperature changes which they produce cannot be equalized by conduction. The compressions and rarefactions are so nearly adiabatic that the adiabatic coefficient of elasticity must be used. For liquids and solids, the difference between the isothermal and adiabatic coefficients of elasticity is negligible. For gases, the difference cannot be neglected. For monatomic gases, the ratio between the adiabatic and the isothermal coefficients of bulk modulus is 1.66; for diatomic gases, it is between 1.40 and 1.41; for triatomic molecules, it is between 1.30 and 1.31. For air, it is 1.404.

20. The Velocity of Sound in a Gas. — If a change of pressure of ΔP dynes per square centimeter acting upon a body of volume V cubic centimeters reduces the volume by ΔV cubic centimeters, then the

$$\text{bulk modulus}\left[=\frac{\text{stress}}{\text{strain}}\right] = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{V\Delta P}{\Delta V}\cdot \tag{79}$$

If the body be a perfect gas of volume V when under a pressure P, and volume $(V - \Delta V)$ when under a pressure $(P + \Delta P)$, then when temperature is constant we will have from Boyle's law,

$$\begin{aligned} PV &= (P + \Delta P)(V - \Delta V) \\ &= PV - P\Delta V + V\Delta P - \Delta P\Delta V. \end{aligned}$$

Now, since ΔP and ΔV are small compared with P and V, the product $\Delta P\Delta V$ is negligible compared with P and V. Hence, the above equation reduces to

$$V\Delta P = P\Delta V,$$

or,

$$\frac{V\Delta P}{\Delta V} = P. \tag{80}$$

Comparing (79) with (80), we see that for perfect gases the isothermal bulk modulus equals the pressure.

Since, for air, the adiabatic bulk modulus equals 1.404 times the isothermal bulk modulus (Art. 19), the velocity of sound in air is (78) and (80),

$$v\left[= \sqrt{\frac{E}{\rho}}\right] = \sqrt{\frac{1.404\,P}{\rho}}. \tag{81}$$

From this equation we can deduce three important results:

(*a*) Since, from Boyle's law, when a perfect gas is at constant temperature P/ρ is constant, the above equation shows that when a gas is at constant temperature, an alteration of pressure does not affect the velocity of sound.

(*b*) In different gases at the same temperature, the velocity of sound varies inversely as the square root of the densities.

(*c*) If a given mass of gas maintained at constant pressure have a density ρ_0 and volume v_0 when at the temperature 0° C., and a density ρ_t and volume v_t when at the temperature t° C., we have from (81),

$$\frac{v_t}{v_0} = \frac{\sqrt{\rho_0}}{\sqrt{\rho_t}}.$$

And since so long as the mass remains constant the density varies inversely as the volume,

$$\frac{\rho_0}{\rho_t} = \frac{1 + \alpha t}{1},$$

where α is the temperature coefficient of cubical expansion.

Whence,
$$\frac{v_t}{v_0}\left[=\frac{\sqrt{\rho_0}}{\sqrt{\rho_t}}\right]=\sqrt{1+\alpha t}, \tag{82}$$

or, for air,

$$\begin{aligned} v_t &= 331.29\sqrt{1+0.00366\,t} \text{ meters per sec.} \\ &\doteqdot 331.29\,(1+\tfrac{1}{2}\,0.00366\,t) \\ &\doteqdot 331.29\,(1+0.00183\,t) \\ &\doteqdot 331.29+0.6\,t \text{ meters per second.} \end{aligned} \tag{83}$$

21. Velocity of a Transverse Wave along a Flexible String. — Fig. 11 represents a hump travelling along a flexible string with velocity v. Let us consider the forces acting on an element ab of mass M and of such a short length l that the element is an arc of

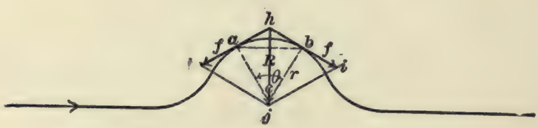

FIG. 11.

a circle of radius r. While the hump is moving along the string, the hump includes different portions of the string. But at any instant, the hump includes an element of length l and of mass M which is moving in a circular arc with speed v. For this motion in a circle of radius r, there must act upon the element a force directed toward c having a magnitude

$$F=\frac{Mv^2}{r}.$$

In the present case, this force is due to the resultant of the two equal tensions f acting at the ends of the element. The magnitude of this resultant R will now be found.

From the figure, the triangles hij and abc are similar because the corresponding sides are perpendicular. Whence,

$$\frac{hj}{hi}\doteqdot\frac{ab}{bc}, \quad \text{or,} \quad \frac{R}{f}\doteqdot\frac{l}{r},$$

or,
$$R=\frac{fl}{r}.$$

Consequently,

$$\frac{Mv^2}{r} = \frac{fl}{r}.$$

$$v = \sqrt{\frac{fl}{M}}.$$

If we represent the mass per unit length of the string by *m*,

$$v = \sqrt{\frac{f}{m}}. \tag{84}$$

22. Transverse Stationary Undulation of a Flexible String. — If a stretched flexible string be plucked, the plucked element will vibrate transversely with simple harmonic motion. This periodic motion by being handed on successively from one element of the string to the next will produce a transverse wave. This wave is reflected from the ends of the string. The two superposed waves produce a motion of the string which is called a stationary undulation or standing wave. There are certain points one-half wave-length apart which remain at rest. These points are called nodes. The segment of the string between two adjacent nodes vibrates back and forth, all points of the segment passing through the equilibrium position at the same instant.

Now during the time that a particle makes one complete vibration, the wave which it sets up advances one wave-length. If a particle vibrate n times per second, the wave which it sets up will advance in each second a distance $n\lambda$, where λ represents the wave-length. Hence the velocity of the wave is

$$v = n\lambda.$$

Substituting in this equation the value of the velocity of a transverse wave in a stretched string (83), we have for the frequency of a stretched string,

$$n\left[= \frac{v}{\lambda}\right] = \frac{1}{\lambda}\sqrt{\frac{f}{m}}, \tag{85}$$

where f is the force stretching the string and m is the mass of unit length of the string.

Exp. 55. Velocity of Sound in Air by Means of a Resonance Tube

THEORY OF THE EXPERIMENT. — Read Arts. 15, 17, 19, and 20. When a vibrating tuning fork is placed at the mouth of a tube closed at the other end, the air in the tube will be set into stationary undulation if a pulse travels twice the length of the tube while the tuning fork makes an odd number of half vibrations. When the air within the tube is in stationary undulation, there is a series of nodes one-half wave-length apart, the first node being situated at about one-quarter wave-length from the open end of the tube, and the last at the closed end of the tube.

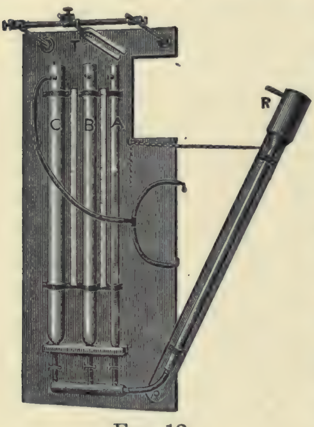

FIG. 12.

After reflection at the closed end of the tube, a pulse travels to the mouth where part of its energy is reflected and part emerges into the outer air. The part which is reflected back into the tube is reinforced by another pulse in the same phase due to the vibrating tuning fork. When resonance occurs the sound emitted is very loud.

In this experiment, the length of the tube is varied by means of a movable piston till the tube "speaks" loudly. After noting the position of the piston, the length of the tube is changed till the tube again speaks loudly. The distance the piston was moved is the distance between two nodes, and this equals one-half wave-length of the sound produced by the exciting tuning fork. Knowing the frequency of the tuning fork, the velocity of sound in the air within the given tube is determined by (72).

MANIPULATION. — In the present experiment, the velocity of sound in tubes of three different diameters will be determined. Fig. 12 illustrates a convenient apparatus for measuring the distance between nodes. It consists of three vertical resonance tubes of different diameters. Each tube is provided with a short

side tube near the upper end for the attachment of a rubber tube with ends which can be inserted into the ears of the observer. By means of pinch cocks at the bottom of the three resonance tubes, any one of the tubes can be put into connection with a reservoir R filled with water. By lowering the reservoir, water is allowed to fill the selected tube to any desired amount. The height of water in the tube is indicated by a scale beside the tube. Above the mouths of the resonance tubes is a horizontal rod in which can slide a clamp holding a tuning fork.

With a fork of known frequency over the mouth of the first resonance tube, and the tips of the listening tubes in the ears, let water into the resonance tube till a region of maximum loudness is found. One will find, as one would expect from a consideration of Art. 17, that maximum resonance does not occur at a definite point, but that there is no appreciable change of loudness for a displacement of the water level of several millimeters. Note the height of the water when the sound appears loudest. In the same manner, by letting in more water, find as many places of maximum loudness as occur within the length of the resonance tube. Note the temperature.

Do the same with the two other resonance tubes.

Take similar observations with all three resonance tubes, using forks of different frequencies.

Compute the velocity of sound in each tube for each frequency used. By means of (82), compute the velocity at 0° C.

Exp. 56. Velocity of Sound in Solids by Kundt's Method

Theory of the Experiment. — Read Arts. 15, 17, and 20. Representing by v_1 the velocity in a given solid of a sound of frequency n and wave-length λ_1, we have (72)

$$v_1 = n\lambda_1.$$

Similarly, the velocity in air of a sound of wave-length λ_2 and the same frequency may be written

$$v_2 = n\lambda_2.$$

Whence,

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}. \tag{86}$$

In Kundt's method the wave-lengths of a sound in two media are obtained from stationary undulations. Consider a rod clamped rigidly at two points each a quarter of its length from an end. If the middle section of the rod be stroked longitudinally with a piece of rosined leather, the rod will be set into longitudinal vibration. There will be nodes at the fixed points and antinodes at the free ends. Thus, the wave-length of the vibration equals the length of the rod.

If one end of the vibrating rod project into the open end of a straight glass tube whose length can be adjusted by means of a piston, it is possible to so adjust the length of the air column that it will be set into resonant forced vibration of the frequency of the vibrating rod. The air within the tube will now be in stationary undulation. At the nodes the amplitude of vibration of the air is minimum. At the antinodes it is maximum.

Kundt located the nodes by means of the effect of the vibrating air within the tube on a narrow line of light powder along the bottom of the tube. At the antinodes the dust will be set into turbulent motion, whereas at the nodes the dust will be almost quiet. If the tube be twisted about its axis till the line of powder is a little above the lowest element of the tube, the dust will remain stationary at the quiet places and fall at the places of turbulence. The wave-length of the sound in the tube equals double the distance between two consecutive nodes.

Manipulation. — Before beginning the experiment, pour out of the wave tube the dust left by the previous experiment and clean the tube by pushing through it a closely fitting wad of cotton or clean waste. Dry the tube by heat.

For indicating the position of the nodes use either lycopodium powder or cork filings. Whichever sort of dust is employed should be dried before use. Pour into the wave-tube sufficient of the dust to form a streak not more than two millimeters wide along the entire length of the tube.

Arrange the wave tube and sounding rod as shown in Fig. 13, taking care that the disk on the end of the sounding rod does not touch the wave tube. Tap the tube with a piece of wood so as to bring all of the dust in line. Twist the tube about its axis till the line of dust is just on the verge of slipping down to the bottom.

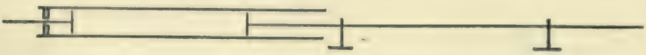

FIG. 13.

Now set the rod into longitudinal vibration by chafing it with a piece of rosined leather. If distinct nodes are not formed, change the length of the air column by moving the piston about one centimeter, again arrange the line of dust, and again chafe the rod.

When distinct nodes are formed, measure the length of the air column, and divide this distance by the number of segments into which the line of dust is divided. Also measure the length of the sounding rod.

From these values the ratio of the speed of sound in the rod to the speed in the air of the tube can be computed by means of (86). Reduce the value thus obtained to the speed at 0° C. by means of (82).

Exp. 57. Determination of the Ratio between the Velocities of Sound in Two Gases by Kundt's Method

THEORY OF THE EXPERIMENT. — Read Arts. 15, 17, 20, and Exp. 56. Stationary undulations of the same frequency can be set up in two different gases simultaneously by means of Kundt's double

FIG. 14.

tube apparatus. This consists, Fig. 14, of two glass tubes each closed at one end by a tightly stretched thin rubber membrane, and at the other end by a movable piston. The rubber membranes are in contact with disks fastened to the ends of the vibrating rod R.

Representing the velocities of the sound in the two tubes by v_2 and v_3, and the corresponding wave-lengths by λ_2 and λ_3, we have, since the frequency is the same,

$$\frac{v_2}{v_3} = \frac{\lambda_2}{\lambda_3}. \tag{87}$$

MANIPULATION. — Fill the tubes with the assigned gases and proceed as in Exp. 56.

Exp. 58. Study of the Laws of Vibrating Strings by Melde's Method

THEORY OF THE EXPERIMENT. — Read Arts. 15, 18, 21, and 22. From (85) it is seen that the frequency of a transversely vibrating stretched string is: (*a*) inversely proportional to the length of the wave in the string; (*b*) directly proportional to the square root of the tension; (*c*) inversely proportional to the square root of the mass of unit length of the string. The object of this experiment is to illustrate these laws of vibrating strings.

It would be possible to find the relation between the frequency, wave-length, tension, and mass of string per unit length, by varying these quantities one at a time, while the other three were kept constant. But as it is rather difficult to measure the frequency, we shall keep the frequency constant throughout this experiment and vary the other quantities. The method is to measure the tensions required to cause strings of various masses per unit length to vibrate with stationary undulations of different wave-lengths while the frequency of the vibrations of the string remains constant. Substituting these various observed values of f, m, and λ in (85), the frequency for each case is to be computed. If the work is accurately performed, all of the computed values of the frequency will be the same The experimental methods here to be used, however, are not susceptible of the highest precision.

MANIPULATION. — The apparatus, Fig. 15, consists of a flexible string which is set into stationary undulation by means of an electrically driven tuning fork. The tension of the string is

varied by changing the mass of shot in a little bucket suspended from one end of the string.

With a small tension the entire string, from the tuning fork to the pulley or fret, will vibrate in a single segment. By increasing the tension, the number of segments may be increased at will. The length of one segment, that is, the distance between two

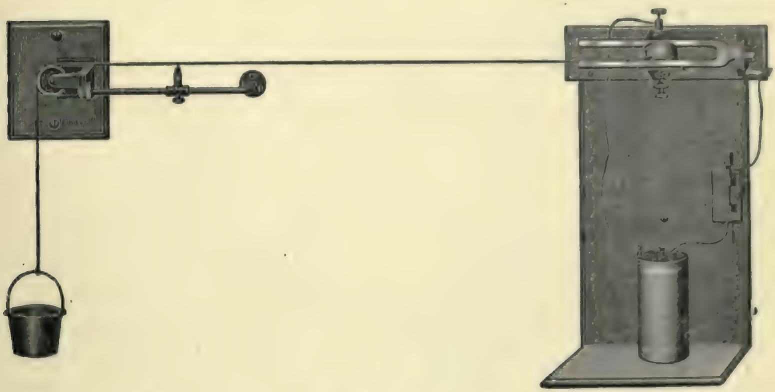

FIG. 15.

consecutive nodes, is one-half wave-length. The wave-length can be adjusted either by changing the tension of the string or by changing the length of the vibrating portion of the string by means of the fret shown under the string in the figure.

In this experiment three strings of different mass per centimeter are to be used. Before attaching any one of the strings to the tuning fork, weigh and measure the length of each.

For each string find the tensions which must be applied in order that it will vibrate in 4, 6, and 8 segments.

Expressing tensions in dynes, wave-length in centimeters, and mass per unit length in grams per centimeter, compute, by means of (85), the frequency of vibration of the string for each of the nine cases.

Exp. 59. Study of the Laws of Vibrating Strings by Lissajous Figures

THEORY OF THE EXPERIMENT. — Read Arts. 8, 15, 18, 21, and 22. The object of this experiment is to illustrate the laws of transversely vibrating strings expressed in (85). The method consists in keeping constant two of the four quantities, n, λ, f, and m, and finding how a third is varied when the fourth is changed.

By adjusting the length and tension of the string, the frequency of vibration can be brought into known relation with the frequency of an electrically driven tuning fork. A comparison of the frequencies is made by means of Lissajous figures as follows: Over the middle of the string is mounted the tuning fork, Fig. 16, to one prong of which is attached the objective lens of a low power microscope. The eyepiece of the microscope is separate and is mounted on a rigid support. The vibration of the tuning fork is in a line perpendicular to that of the vibration of the string. Fastened on the string immediately below the objective lens is a brightly illumined speck of mercury or of chalk. On looking through the eyepiece one sees a Lissajous figure traced by the image of the bright speck. If the figure does not change in form, the relation between the frequencies of the two component vibrations can be obtained by means of the rule stated in the last paragraph of Art. 8.

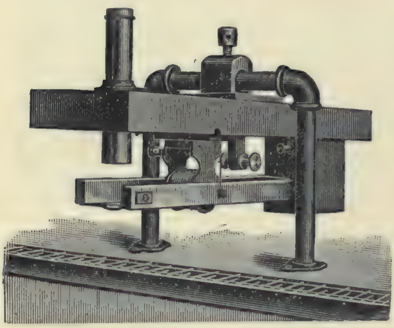

FIG. 16.

MANIPULATION. — Throughout the present experiment a single string will be used. Consequently, m, the mass per unit length, will be constant. Also, throughout the present experiment, the string is to vibrate in a single segment. This requires that the string be always plucked near the middle. The wave-length, λ, will equal two times the length of the vibrating part of the string.

Various values of λ will be obtained by altering the length of the vibrating segment by means of moving frets.

During the experiment the string will be caused to vibrate with three different frequencies. The fork has an unknown constant frequency. It will be convenient to use as ratios of the frequency of the string to that of the fork, the ratios 1 : 1, 2 : 3, and 3 : 4. The corresponding Lissajous figures are given in Fig. 4.

(*a*) With m and n constant, find values of f corresponding to various values of λ. With the frets 100 cm. apart, adjust the tension by varying the amount of shot in the little bucket on the end of the string till on plucking the string near the middle point the Lissajous figure is an ellipse of fixed form. The frequency of the string now equals that of the fork.

Repeat with two other distances between the frets.

Expressing f in dynes and λ in centimeters, find the numerical value of $\sqrt{f} \div \lambda$.

(*b*) With m and λ constant, find values of f corresponding to three different values of n. With the frets separated by any convenient fixed distance, first, adjust the tension till the Lissajous figure indicates that the string and the fork are vibrating with the same frequency. Then, with the frets unchanged, adjust the tension till the Lissajous figure shows that the ratio of the frequency of the string to that of the fork is 2 : 3. Again, adjust the tension till the ratio is 3 : 4.

From these data, compute the relation between the first and the second value of $\sqrt{f} \div n$, and also that between the first and the third value.

(*c*) With m and f constant, find the values of λ corresponding to three different values of n.

From these data compute the ratio of the first to the second value of λn, and also that between the first and third.

Exp. 60. Comparison of the Frequencies of Two Forks of Nearly the Same Pitch by Means of Beats

Theory of the Experiment. — Read Art. 16. When two notes of nearly the same pitch are sounded together, beats will be produced. The number of beats per second equals the difference

between the frequencies of the two notes. The method of beats is the one usually employed by musicians in tuning instruments. It is not suited to the comparison of high notes on account of difficulty in distinguishing the beats. Nor is it satisfactory for the comparison of two notes which give less than two or three beats per second on account of difficulty in deciding at what instant the loudness of the sound is maximum or minimum.

Beats are easily heard in the case of two vibrating tuning forks, or two wires. They are not so readily observed in the case of a tuning fork and a wire.

MANIPULATION. — Actuate each of the tuning forks by striking each lightly with a small felt-faced mallet or a piece of sole leather. Count the number of beats for a period of about sixty seconds and by means of a stop watch measure the interval of time between the middle of the first and the middle of the last beat. This time, in seconds, divided by the number of beats diminished by one, gives the number of beats per second. This number equals the difference in the frequencies of the two tuning forks.

If one cannot distinguish which fork is of the higher pitch, one can determine it as follows: Attach a small bit of beeswax to the end of one prong of one of the forks. The pitch of that fork will be thereby lowered. Determine the number of beats per second as before. Fewer beats per second would show that it was the fork of higher pitch which had been loaded. More beats per second, however, would be produced if either the fork of lower pitch had been slightly loaded or the fork of higher pitch had been loaded so much as to make it lower in pitch than the other. To distinguish between these two possibilities, diminish the size of the bit of beeswax and again note the number of beats per second.

Exp.[3] 61. Determination of the Frequency of a Tuning Fork by A. M. Mayer's Method

THEORY OF THE EXPERIMENT. — In this method the fork under examination traces a wavy line on a piece of smoked paper fastened to a rotating cylinder, and an electric spark passes from the tracing point to the cylinder at every clock beat. Thus the frequency of

the fork, when loaded with the stylus, is given by the quotient obtained by dividing the number of vibrations traced on the smoked paper between two punctures by the interval of time between the punctures. The alteration of frequency due to the addition of the stylus is obtained from the difference between the number of beats produced when the fork with the stylus is sounded at the same time with a second fork of nearly the same frequency, and the number of beats produced when the fork under examination without the stylus is sounded at the same time with the second fork.

The cylinder carrying the smoked paper is mounted on an axle provided with a screw thread such that when the cylinder is rotated through one revolution, the cylinder advances about 0.5 cm. in the direction of its length.

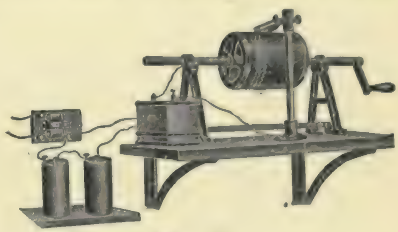

FIG. 17.

The time-marking device consists of a clock pendulum which once every beat completes an electric circuit containing a galvanic cell and the coil of a telegraph relay. The relay circuit includes a battery and the primary of a small induction coil. One terminal of the secondary of the induction coil is connected to the tuning fork, and the other terminal to the frame carrying the revolving cylinder. Thus at every clock beat a spark punctures the blackened paper.

MANIPULATION. — Stretch tightly about the metal cylinder a sheet of glazed paper, being careful to make a smooth joint where the two edges are fastened together. Now place a candle flame under one end of the cylinder and slowly rotate the cylinder till the entire surface is coated with soot. Since the paper is backed by a good heat conductor, there will be little danger of scorching the paper even though the flame is in contact with it.

Wrap about one prong of the fork a few turns of fine stiff wire, leaving one end projecting for use as a stylus or tracing point. No. 36 wire of manganin, german silver, or hard-drawn brass is suitable for this purpose. Attach the wire to the fork by a drop

of sealing wax. The marking point must be smooth. If it be not smooth it may be made so by touching it with a small blowpipe flame.

Mount the tuning fork so that the stylus presses lightly on the smoked paper. Arrange the time-marking device, bow the fork, and rotate the cylinder. Repeat the bowing whenever necessary. After the cylinder has advanced the whole length of the sheet of paper, disconnect the marking device and remove the fork. By means of an atomizer, spray the smoked surface with alcohol containing a very small amount of shellac.

Count the number of vibrations included within some even number of clock beats. The reason for taking an even number of clock beats is to eliminate any error due to an inequality of the beats in opposite directions. From these data compute the frequency of the fork with the attached stylus. This value is in slight error due to the friction between the paper and the stylus. Except when very precise results are required, however, this error is quite negligible.

The addition of the stylus diminishes the frequency of the fork under investigation. The correction to be added is made by the aid of a second fork of unknown higher pitch as follows: By adding wax near the ends of the prongs of the secondary fork, adjust the frequency till about five or six beats per second occur when the secondary fork and the fork with the stylus are sounded together. Knowing the frequency of the fork with the stylus, the frequency of the secondary fork is obtained.

Now remove the stylus from the fork under investigation and observe the number of beats produced per second when the two forks are sounded together. From this result, the required frequency of the fork under investigation is obtained.

Exp. 62. Determination of the Frequency of an Electrically Driven Tuning Fork by Means of a Phonic Wheel

Theory of the Experiment. — Read Art. 18. La Cour's phonic wheel consists of a hollow cylinder capable of rotation about a vertical axis. To the convex surface of the wheel is fastened at

equal intervals a number of iron bars parallel to the axis of the wheel. In front of the wheel is an electromagnet so arranged that when the magnet is energized by an electric current, the nearest iron bar is pulled toward the poles of the magnet. If the current be made and broken at regular intervals, the wheel will rotate continuously. The wheel is filled with mercury, which, by producing a drag on the inside surface, tends to keep the speed uniform. A revolution-counter geared to the shaft of the wheel indicates the product of the number of revolutions of the wheel and the number of iron rods on the periphery.

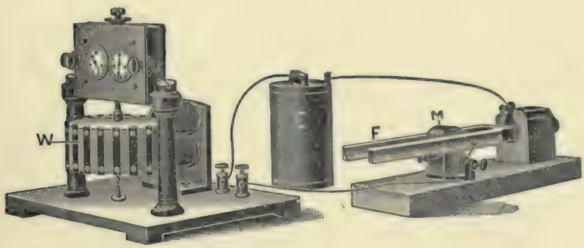

FIG. 18.

On connecting an electrically driven tuning fork in series with a galvanic cell and the electromagnet of the phonic wheel, a current will energize the electromagnet once during each complete vibration of the tuning fork. Thus, the wheel will rotate the distance between two consecutive bars during every complete vibration. Consequently, the number of complete vibrations made by the tuning fork while the wheel makes any given number of complete revolutions is indicated directly on the scale of the instrument. For example, with a particular tuning fork marked "100," the scale reading changed from 38,980 to 41,440 in 50¼ seconds. Consequently, the number of double vibrations made per second was

$$\frac{41{,}440 - 38{,}980}{50.2} = 49.0,$$

or, since the frequency of tuning forks is usually expressed in single vibrations per second, the frequency of the fork was 98.

Manipulation. — After connecting the apparatus, start the tuning fork by striking one prong lightly with a felt-faced mallet. Start a calibrated stop-watch at the instant the lower index hand crosses a given line of the scale and note the scale reading. After an interval of about one minute, stop the watch when the same index crosses a given line of the scale, and note the scale reading and the watch reading.

Take ten similar sets of data.

Exp. 63. Determination of the Frequency of a Tuning Fork by Means of a Vibration Microscope and Phonic Wheel

Theory of the Experiment. — Read Arts. 8 and 18 and Exp. 62. The vibration microscope consists of an electrically driven tuning fork on one prong of which is the objective of a microscope. The eyepiece of the microscope, M, is mounted on

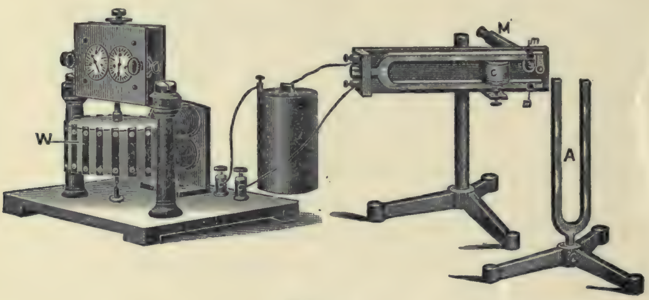

Fig. 19.

the frame of the instrument. By means of two masses, mm, which can be moved along the prongs, the frequency of the fork can be varied. The magnet coil, C, is in series with a galvanic cell and the coil of the phonic wheel as shown in Fig. 19.

The tuning fork, A, under examination, is placed in front of the microscope M so that the planes of vibration of the two forks are at right angles to one another. On looking through the eyepiece toward the image of a particle of mercury or chalk attached to one prong of the fork under test, one sees a Lissajous figure. By

varying the position of the adjusting masses, *mm*, the Lissajous figure can be changed to a form that can be recognized as being the resultant of components having frequencies in known ratio to one another.

The frequency of the vibration microscope can be found by means of the phonic wheel in the manner described in Exp. 62. The required frequency of the fork under examination can then be computed.

MANIPULATION. — Fasten a particle of mercury to one prong of the fork under examination by means of wax and vaseline. Focalize the microscope on the particle of mercury. Set the two forks into vibration and observe the Lissajous figure. Move the adjusting masses on the vibration microscope till the Lissajous figure is a simple one which changes in form very slowly. Figures having components in the ratio 1 : 1, 1 : 2, 1 : 3, or 1 : 4 are readily recognized.

By means of a stop-watch find the time occupied by the figure in going through one complete cycle. If this time be less than ten seconds, readjust the masses on the vibration microscope till the time is more than ten seconds. Record this time.

By the method of Exp. 62, find the frequency of the vibration microscope.

One must now determine whether the correction to be made, on account of the fact that the Lissajous figure does not remain of fixed form, should be added or subtracted. As an example of the method, suppose that the frequency of the vibration microscope is found to be 62.5 complete vibrations per second, and that the Lissajous figure is that of two components having frequencies approximately in the ratio 1 : 4. Also, suppose that the figure goes through one complete cycle of changes in 11.4 seconds. Then, the ratio of the frequency of the vibration microscope to that of the fork under examination is either

$$1:\left(4+\frac{1}{11.4}\right), \quad \text{or,} \quad 1:\left(4-\frac{1}{11.4}\right).$$

In order to distinguish between these two possibilities, attach a small bit of beeswax to the end of one prong of the fork under

examination. Observe the time of one complete cycle of the figure. If this time be longer than that before the wax was attached, it follows that by diminishing the frequency of the fork under test, the ratio of the frequencies of the two forks has been made more nearly equal to 1 : 4. That is, the frequency of the fork under examination was originally too great to make the frequency of the vibration microscope and the frequency of the other fork in the ratio 1 : 4. Hence, before the wax was attached the frequencies of the two forks were in the ratio

$$1 : \left(4 + \frac{1}{11.4}\right) = 62.5 : n,$$

where n is the required frequency of the fork under examination.

If, however, the loading of the fork under test gives a briefer cycle, then the fork under test originally may have been either so high in pitch as to give the ratio 1 : 4+, or so low as to give the ratio 1 : 4−. In the latter case, the loading would cause the ratio to depart still more from 1 : 4 and the duration of the cycle would diminish. In the former case, namely, when the ratio is 1 : 4+, a cycle of longer duration would also result if the loading made the ratio less near 1 : 4 than before. This would occur if the ratio were reduced to a sufficiently small value 1 : 4−. By noting the effect on the duration of the cycle produced by diminishing the mass of wax attached to the fork, one can determine whether the original ratio was 1 : 4+ or 1 : 4−.

Exp. 64. Determination of the Frequency of a Tuning Fork by Means of a Clock-Fork

THEORY OF THE EXPERIMENT. — Read Art. 8. An ordinary clock consists of a pendulum, a device to maintain the pendulum in vibration, and a device to count the number of vibrations made by the pendulum. The clock-fork, or tuning-fork chronoscope, consists of a tuning fork, a device to maintain the tuning fork in vibration, and a device to count the number of vibrations made by the tuning fork. Usually a fork is employed which makes 64 complete vibrations per second. The frequency of the fork may be varied by moving masses attached to the prongs.

The face of the clock is provided with scales for reading hours, minutes, seconds, and vibrations. When the fork makes exactly 64 complete vibrations per second, the clock keeps accurate time. An eyepiece attached to the frame of the clock and an objective lens attached to one prong of the fork permit the instrument to be used as a vibration microscope for the production and observation of Lissajous figures.

When used for rating a tuning fork, the fork under examination T is mounted in front of the instrument with the plane of vibration perpendicular to the plane of vibration of the clock-fork F, Fig. 20. On viewing through the eyepiece, E, the image of a speck of chalk dust on the end of the vibrating fork under examination, one sees a Lissajous figure. From the shape and the rate of change of this figure, the ratio of the frequencies of the two forks can be determined with greater precision than by any other method.

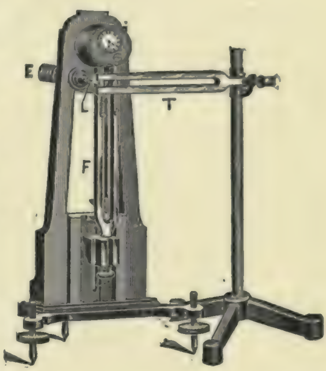

FIG. 20.

The method of rating a tuning fork is illustrated in the following example. Suppose that the fork under examination is marked D_3. It should give 290.33 complete vibrations per second. That is, the ratio of the frequencies of the clock-fork and the fork under examination is nearly 1 : 5. The clock-fork will now be adjusted, by moving the attached masses, till the ratio of the frequencies becomes as near as possible to 1 : 5.

If the fork under examination makes 290.33 vibrations per second, then in order that the ratio of the frequencies may be 1 : 5, the clock-fork must be adjusted to give $290.33 \div 5 = 58.066$ vibrations per second. That is, the clock must be regulated to run at $\frac{58066}{64000}$ of its normal rate. And since there are 86,400 seconds in a day, the clock-fork must be adjusted to indicate $(\frac{58066}{64000})$ 86,400 [= 78,389] seconds per day.

Suppose that the clock-fork has been adjusted so as to indicate 78,389 seconds per day. If the other fork makes 290.33 vibrations per second, then the Lissajous figure will not vary in form. If the figure does go through cyclic changes in form, then the frequency of the fork under examination is not 290.33.

Suppose that the figure goes through a cycle of changes in 7.3 seconds. Then, the frequency of the fork must be $290.33 \pm \frac{1}{7.3} =$ 290.45 or 290.21. To determine which value to use, that is, whether the fork is above or below the pitch it is supposed to have, attach a small piece of wax to one prong and again note the period of a cycle. If the fork were flat previously, the addition of the wax will increase the period of the cycle. If the fork were sharp previously, the addition of the wax will decrease the period of the cycle. Suppose the addition of the wax shows that the fork was originally sharp, then the value 290.45 is the one to be selected.

To adjust the frequency of the clock-fork to an exact predetermined value is tedious and unnecessary. If, in the above example, the clock-fork were adjusted till it indicated not 78,389 seconds in one day, but 78,380 seconds in one day, then the frequency of the fork under investigation is $(290.45)\ \frac{78380}{78389} = 290.42$ double vibrations per second.

Manipulation. — Find the nearest simple ratio that is approximately equal to the ratio of the frequency of the clock-fork to that of the fork under examination. If the resultant of the vibrations of the two forks be a Lissajous figure of unfamiliar form, plot the figure by the method of Exp. 51.

Adjust the positions of the masses on the clock-fork till this Lissajous figure is produced. It is unnecessary to continue the adjustment till the figure remains constant in form, but the adjustment should be continued till the time occupied by a cycle is not less than ten seconds. Find the period of a cycle with a stop-watch. Find the rate of the adjusted clock-fork from a pair of simultaneous readings of the clock-fork dial and a chronometer taken several hours apart.

CHAPTER III

HEAT

23. Temperature. — The comparison of temperatures involves several arbitrary conventions. Temperatures cannot be directly measured, — they can be compared only in terms of some other phenomenon which depends upon temperature. Of the various phenomena which are used for the comparison of temperatures, the following are the most important: (*a*) change of the volume of a gas or liquid kept at constant pressure, (*b*) change of the pressure of a gas kept at constant volume, (*c*) change of the electric resistance of a metal wire, (*d*) production of an electromotive force at the junction of two dissimilar metals, (*e*) quantity of energy radiated by the hot body, (*f*) luminous intensity of the radiance of a particular color emitted by the hot body. In cases (*a*), (*b*), (*c*), and (*d*) it is necessary to select a particular thermometric substance. In all cases it is necessary to adopt two particular temperatures as standard or fixed points of a thermometric scale, and to divide the interval between these points into a definite number of spaces or degrees.

The scale of temperatures that has been adopted as standard is based on the change of pressure which a change of temperature produces in a fixed mass of hydrogen kept at constant volume. By means of such a thermometer, temperatures as high as 1700° C. can be compared. However, as this standard or normal thermometer is both bulky and fragile, and requires careful manipulation and considerable computation in making a temperature determination, it is seldom used except in scientific work and for the purpose of standardizing other thermometers. All other thermometers are calibrated in terms of the normal thermometer.

On account of the comparatively simple technique necessary in its use, the mercury-in-glass thermometer is employed when-

ever the conditions of the measurement permit. By using a very hard glass, and filling the space above the mercury with an inert gas at a pressure sufficient to prevent boiling of the mercury, a mercury-in-glass thermometer can be used up to about 550° C. (1000° F.). Mercury-in-quartz thermometers having the space above the mercury filled with gas at 60 atmospheres pressure can be used up to 700° C.

Thermometers available for temperatures above 500° C. are often called *pyrometers.* The electric resistance and the thermoelectric instruments are available for temperatures up to about 1500° C. For temperatures above this, radiation and optical pyrometers are available.

Measurements of temperature, even by means of the mercury-in-glass thermometer, are subject to so many sources of error that an accurate determination of temperature is a task of some difficulty. Nevertheless, the thermometric methods have become so highly developed that if proper precautions are taken and proper corrections made, a measurement of temperature made with a mercury-in-glass thermometer between 0° C. and 100° C. can be trusted to 0°.005. The methods described in the following pages correspond to an accuracy of about 0°.05.

The principal sources of error in the use of a mercury-in-glass thermometer are: —

(*a*) Errors in reading the thermometer due to parallax. Usually the scale of a thermometer is at some distance in front of the capillary, so that, unless the line of sight is normal to the length of the tube, the reading is too high or too low. The two principal methods employed for keeping the line of sight normal to the length of the thermometer tube are, (*a*) to hold a small mirror against the back of the thermometer, and to place the eye in such a position that the top of the mercury thread is in line with the image of the eye seen in the mirror; and (*b*) to observe the thermometer at a distance by means of a telescope containing a cross-hair in the eyepiece, the telescope being fastened normal to a rod placed parallel to the thermometer tube. The telescope must be arranged so that when it is moved along the supporting

rod to observe the end of the moving column at different heights, it will always remain normal to the supporting rod. A cathetometer is usually most convenient for this purpose, but a short open tube without lenses, having cross-hairs at the two ends, and sliding either on the thermometer itself or on a parallel rod, serves the purpose very well.

(*b*) Errors due to the changes in the volume of the bulb lagging behind the changes in temperature. A rising thermometer indicates too low, and a falling thermometer too high, a temperature. This lag is due to the viscosity of the glass of which the thermometer is made. If a thermometer be kept for some days at a uniform temperature of, say, 20° C., and then plunged into a bath of melting ice and the temperature observed; and if it then be heated to a temperature of 100° C., and again plunged into the bath of melting ice, the temperature now observed will be lower than the one previously obtained. The increase in the volume of the bulb due to the high temperature does not at once disappear, and the zero point may be depressed as much as half a degree for some kinds of glass. This depression of the zero point is greater when the temperature to which the thermometer has been raised is greater, and when the time is greater that the thermometer is kept at the higher temperature. The depression persists for weeks and even months before the normal volume of the bulb is regained. It follows that while the thermometer is being used at various temperatures the zero point is constantly changing. This makes no temperature determinate unless the value of the zero point at this particular time is known. The value of the zero point can be obtained by cooling the thermometer down to the temperature of melting ice immediately after the desired temperature reading has been made. Then, if no other errors affect the observation, the true temperature is the difference between the observed temperature and the value of the depressed zero. This is called the "depressed zero method" of measuring temperature, and is the only method capable of yielding the most accurate results attainable.

(*c*) Errors due to the exposed column of the thermometer being at a temperature different from that of the bulb.

Let T denote the true temperature of the bulb;
t, the temperature indicated by the thermometer;
s, the temperature of the exposed part of the stem; and
e, the reading where the stem emerges from the bath.

Then the length of the exposed column is $(t - e)$ degrees, and the difference between its temperature and that of the bulb is $(T - s)$. Since the coefficient of apparent expansion of mercury in glass is about 0.000156 per degree C., this exposed part of the column, if it were to be raised in temperature $(T - s)$ degrees, would increase in length $0.000156\,(t - e)\,(T - s)$ degrees. That is,

$$T = t + 0.000156\,(t - e)\,(T - s).$$

The difference between T and t is so small compared with $(T - s)$ that the substitution of t for T will produce an inappreciable error. Whence,

$$T \doteq t + 0.000156\,(t - s)\,(t - e). \tag{88}$$

(*d*) Errors due to inequalities in the bore of the tube. These errors are corrected by calibrating the tube as described in Experiment 66.

(*e*) Error in the graduation of the stem; that is, although the divisions are of equal length, their length is not such as to make just a hundred divisions between the boiling point and the freezing point of water. Let T_v denote the true temperature of the vapor above boiling water as determined by reading the barometer (Art. 27 and Table 3), t_v the temperature indicated by the thermometer when it is immersed in the vapor above boiling water, and t_0 the depressed zero reading taken immediately after t_v was observed. Then the number of degrees that ought to be between the point where the thermometer reads t_0 and that where it reads t_v is T_v, and the number of degrees that really are between those points is $(t_v - t_0)$. It follows that any temperature difference read from the thermometer is to be multiplied by a factor

$$k = \frac{T_v}{(t_v - t_0)}. \tag{89}$$

(*f*) Errors due to changes in the pressure to which the bulb is subjected. Any change of pressure will cause a change of the height of the mercury column independent of any change of temperature. Usually the experimental method can be arranged so as to eliminate this source of error.

(*g*) Error due to capillarity. In a thermometer of very small bore the mercury does not move smoothly but moves in little jumps. This error is much greater when the temperature is falling than when rising. In fact, the capillary action makes it impossible to measure accurately a falling temperature by means of a mercury-in-glass thermometer.

24. The Beckmann Thermometer. — A thermometer designed to estimate temperatures to thousandths of a degree requires such a long space for each degree of scale, that, if constructed on the ordinary plan, the range of the instrument would be limited to a few degrees. This would require the use of a number of instruments to cover the range of ordinary laboratory work. When it is not required to determine definite temperatures but only small temperature differences, the thermometer devised by Beckmann can be used at any temperature for which a mercury-in-glass thermometer is available. The peculiarity of this thermometer is a reservoir *R* at the upper end of the tube (Fig. 21), by means of which the quantity of mercury in the bulb can be increased or diminished. The scale is usually about five centigrade degrees in length and is divided into hundredths of a degree.

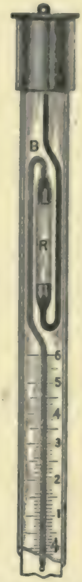

FIG. 21.

In setting the instrument, a sufficient amount of mercury must be left in the bulb and stem to give readings between the required temperatures. First invert the thermometer and tap the tube so that the mercury in the reservoir will lodge in the bend *B* at the end of the stem. Now heat the bulb until the mercury in the stem joins the mercury in the reservoir. Place it in a bath one or two degrees above the upper limit of temperatures to be measured. If now the upper end of the tube be flipped with

the finger, the mercury suspended in the upper part of the reservoir will be jarred down, thus separating it from the thread at the bend B. The thermometer is now set for readings between the required temperatures.

25. Thermal Expansion. — If a solid body has at 0° C. a length l_0 and at $t°$ a length l_t, it is found that the relation between length and temperature is usually expressed very nearly by the equation

$$l_t = l_0 (1 + \alpha t), \tag{90}$$

where α is a constant for any one substance and is called the *coefficient of linear expansion* of that substance. From this equation,

$$\alpha = \frac{l_t - l_0}{l_0 t}. \tag{91}$$

That is, the temperature coefficient of linear expansion is numerically equal to the change in length, per degree, per unit length at 0° C.

Similarly, if any body has at 0° a volume v_0 and at $t°$ a volume v_t, it is found that the relation between volume and temperature is usually expressed very nearly by the equation

$$v_t = v_0 (1 + \beta t), \tag{92}$$

where β is a constant for any one substance and is called the *coefficient of cubical expansion* of that substance.

26. The Two Expansion Coefficients of a Gas. — From (92) the coefficient of expansion of a gas under constant pressure may be written

$$\beta_p = \left(\frac{v_t - v_0}{v_0 t}\right)_p, \tag{93}$$

in which the subscript p indicates that pressure is constant.

If the volume of a gas be maintained constant, the quantity $\left(\frac{p_t - p_0}{p_0 t}\right)$ is constant. This is called the *pressure coefficient,* or the *coefficient of expansion under constant volume.* It may be written

$$\beta_v = \left(\frac{p_t - p_0}{p_0 t}\right)_v. \tag{94}$$

The relation between the magnitude of β_p and β_v will now be found.

In the case of perfect gases it is shown in texts on General Physics that if p, v, m, and T denote respectively the pressure, volume, mass, and absolute temperature of the gas,

$$pv = RmT, \tag{95}$$

where R is a constant which depends only upon the units chosen and not at all upon the nature of the gas nor any other condition. This equation is obtained by combining Boyle's and Charles' laws and is known as the Fundamental Law of Perfect Gases.

Consider a given mass of gas at temperature 0° C. or T_0 absolute, pressure p_0, and volume v_0. When it is heated to t° C., *i.e.*, $(T_0 + t)°$ absolute, let the pressure be represented by p_t and the volume by v_t. From the fundamental law of perfect gases (95),

$$\frac{p_0 v_0}{T_0} = Rm = \frac{p_t v_t}{T_0 + t}. \tag{96}$$

If the volume be kept constant (which is denoted below by the subscript v outside of the parenthesis), the v_t in (96) becomes equal to the v_0, and we have from (96),

$$\frac{1}{T_0} = \left(\frac{p_t - p_0}{p_0 t}\right)_v = \beta_v, \tag{97}$$

or, if the pressure be kept constant,

$$\frac{1}{T_0} = \left(\frac{v_t - v_0}{v_0 t}\right)_p = \beta_p. \tag{98}$$

That is, the coefficient of expansion of a perfect gas at constant pressure is equal to the pressure coefficient of the gas.

27. Reduction of Barometric Readings. — If the Torricellian vacuum above a barometric column were devoid of matter and if there were no capillary force between the mercury and the tube, the weight per unit cross section of a barometric column would equal the pressure of the atmosphere at the place where the barometer is situated. The space above the mercury column is, however, filled with mercury vapor which exerts a small pressure depressing the mercury, and the capillary action between the mercury and

the glass tube also diminishes the height to which the mercury rises.

Again, even if the pressure of the atmosphere does not change, the actual height of the mercury column may be altered in two ways: first, by a change of temperature which not only alters the density of the mercury in the barometer and consequently its height, but which also alters the length of the scale used to measure the height; second, by a change in the force of gravity acting on the mercury as it is moved to different parts of the earth's surface. Consequently, in order that barometric readings taken at different temperatures and at different parts of the earth's surface may be compared with one another, they must be reduced to the heights that would have been observed if the barometer had been at some standard temperature and at some standard position on the earth's surface. The standard conditions arbitrarily selected are the temperature of melting ice and the altitude of the sea level at latitude 45°.

In precise work a barometric reading must be adjusted in the above four particulars, of which two are corrections and two are reductions to standard conditions. The method of making these corrections and reductions to standard conditions will now be considered.

(*a*) *Temperature.*—Let h and ρ represent the observed height and the density of the mercury at $t°$, and let v represent the volume of mass m of mercury at this temperature. Let h_0, ρ_0, and v_0 represent the corresponding quantities at 0° C. Then,

$$\rho g h = \rho_0 g h_0 \quad \text{and} \quad m = v\rho = v_0\rho_0. \tag{99}$$

If β denotes the coefficient of cubical expansion of mercury,

$$v = v_0\,(1 + \beta t).$$

Whence,
$$\frac{v_0}{v} = \frac{1}{1 + \beta t}. \tag{100}$$

And since, from (99), $\dfrac{\rho}{\rho_0} = \dfrac{h_0}{h}$ and $\dfrac{\rho}{\rho_0} = \dfrac{v_0}{v}$,

it follows that
$$\frac{h_0}{h} = \frac{1}{1 + \beta t}. \tag{101}$$

The above equation reduces to

$$h_0 \doteq h\,(1 - \beta t). \tag{102}$$

But the brass scale used to measure the height h is ruled so as to be correct at 0° C. That is, a space on the scale having at 0° unit length has at $t°$ a length $(1 + \alpha t)$, where α is the coefficient of linear expansion of the brass scale. Whence, a distance which at $t°$ is presumably h units long is really $h\,(1 + \alpha t)$ units long. Consequently, the barometric height that would be observed if the barometer were cooled to 0° C. is

$$h_0 \doteq h\,(1 + \alpha t)\,(1 - \beta t),$$

or,

$$h_0 \doteq h\,[1 + (\alpha - \beta)\,t]. \tag{103}$$

Since $\beta = 0.000182$ and $\alpha = 0.000018$ per degree centigrade, (103) becomes

$$h_0 \doteq h\,(1 - 0.00016\,t) \tag{104}$$

if the temperatures are taken in centigrade degrees. If, however, the temperatures are taken in Fahrenheit degrees

$$h_{32} \doteq h\,[1 - 0.00009\,(t - 32)]. \tag{105}$$

(*b*) *Depression due to Capillarity.* — This depends upon the diameter of the bore of the glass tube. Its magnitude may be taken from the following table:

Bore of tube in mm.	2	4	6	8	10
Depression in mm.	2.18	0.70	0.25	0.10	0.04

(*c*) *Reduction to Sea Level at Latitude* 45°. — This is most easily effected by means of Table 2.

(*d*) *Depression due to Pressure of Mercury Vapor.* — This may be neglected except in the most refined work. The values of the vapor pressure of mercury at different temperatures are given in Table 5.

28. Calorimetry. — Calorimetry is the theory and art of measuring quantities of heat. Unfortunately there is no single quantity of heat that is universally adopted as the unit. A common unit in scientific work is the amount of heat required to raise the temperature of one gram of water from 15° C. to 16° C. This

unit is called the 15° calorie, or simply the calorie, or the gram-degree-centigrade thermal unit. In the British system the unit adopted is the amount of heat required to raise the temperature of one pound of water from 60° F. to 61° F. This is called the British thermal unit or the pound-degree-Fahrenheit thermal unit. Throughout this book the calorie will be used exclusively.

The number of thermal units required to raise the temperature of unit mass of a substance from $t°$ to $(t + 1)°$ is called its *thermal capacity* at $t°$. The thermal capacity of a substance is slightly different at different temperatures, but the difference is so minute that except in the most refined measurements it need not be considered. The average thermal capacity of a substance between any two temperatures is the number of heat units required to raise a unit mass of it from one of those temperatures to the other, divided by the difference in the two temperatures. That is, the quantity of heat H required to raise from $t_1°$ to $t_2°$ the temperature of m grams of a substance of average thermal capacity c is

$$H = mc\,(t_2° - t_1°). \tag{106}$$

Throughout the above paragraph it is assumed that between the temperatures considered the body does not melt, solidify, vaporize, nor condense.

When a body does melt, solidify, vaporize, or condense, without changing at all in temperature, the amount of heat absorbed or given out is proportional to the mass of the substance that changes state and depends upon what that substance is. That is,

$$H = mL, \tag{107}$$

where L is a constant called the *heat equivalent** of fusion, solidification, vaporization, or condensation, as the case may be.

* From the fact that the heat absorbed by a body during fusion or vaporization does not change the temperature of the body, it used to be supposed, when heat was considered to be a form of matter, that the heat absorbed during fusion and vaporization existed in the melted or vaporized body in a latent, *i.e.*, a hidden form. This heat was then called the *latent heat* of fusion or vaporization. Now that it is known that heat is a form of energy, viz., that form which changes the temperature of bodies, we prefer to say that the heat absorbed by a body during fusion or vaporization does not exist in the melted body as heat but as some other form of energy. Consequently, the expression *latent heat* is now obsolescent and is giving place to the term *heat equivalent*.

The ratio of the thermal capacity c of a substance to the thermal capacity c_w of water is called the *specific heat* of the substance. Though the thermal capacity of water varies with temperature, this variation is so small that for most purposes it may be neglected. For this reason, throughout the present book c_w will always be taken as unity. It follows that the specific heat of a substance is numerically equal to its thermal capacity. In a loose sense the thermal capacity is often called specific heat, but the student should observe that the same relation holds between these two quantities as between density and specific gravity. The former is a definite physical quantity; the latter is a pure number.

29. The Water Equivalent of a Body. — The mass of water which requires the same amount of heat as a given body in order to change its temperature by the same amount is called the *water equivalent* of the body. Thus, if e represents the water equivalent of a body, and c_w the mean thermal capacity of water between $t_1°$ and $t_2°$, the quantity of heat required to raise from $t_1°$ to $t_2°$ the temperature either of the body or of e grams of water is

$$H = ec_w\,(t_2° - t_1°). \tag{108}$$

Dividing each member of (106) by the corresponding member of (108),

$$e = m\frac{c}{c_w} = ms, \tag{109}$$

where s is the specific heat of the substance. That is, the water equivalent of a body equals the product of its mass and its specific heat.

In determining the water equivalent of a thermometer, only that part of it which changes in temperature is to be considered. This may be taken as somewhat more than the part immersed. Fortunately, the product of the density of mercury by its specific heat is nearly the same as the corresponding product for glass. That is, the water equivalent of a given volume of mercury is about the same as that of the same volume of glass. Since the value of this product is about 0.5 g. per c.c., the water equivalent of a thermometer in grams may be taken as somewhat more than half the volume of the immersed part in cubic centimeters.

Although simple in theory, calorimetric experiments require great care and many precautions. One of the most important sources of error is radiation, *i.e.*, there is a gain or loss of heat because neighboring bodies are at temperatures different from that of the body being studied. The principal methods of diminishing this error are (*a*) to compute the amount of heat actually gained or lost by radiation; (*b*) to determine the temperature which the body would have attained if there had been no radiation; (*c*) to employ a method in which the temperature is kept the same as that of the surroundings.

30. The Radiation Correction. — When a body is at a higher temperature than the surroundings it loses heat to the surroundings, and when at a lower temperature it gains heat from the surroundings. In all calorimetric measurements this fact must receive attention. Three methods for taking account of the heat gained or lost by the body under investigation will now be considered.

(*a*) In *Regnault's Method* is determined the number of heat units gained or lost by the body on account of the surroundings being at a different temperature. It is based on Newton's Law of Cooling. This law may be stated as follows: The rate at which a body cools is proportional to the difference between its temperature and the temperature of its surroundings. If, then, Δt denotes the fall of temperature due to the radiation which occurs in the short time ΔT, and if the temperatures of the body and its surroundings are denoted respectively by t_b and t_s, Newton's law of cooling may also be stated by the equation

$$\frac{\Delta t}{\Delta T} = k' (t_b - t_s),$$

where k' is the proportionality factor. If both members of this equation are multiplied by the water equivalent, e', of the cooling body, then since $e'\Delta t$ is, from (109) and (106), numerically equal to the heat ΔH lost while the temperature falls $\Delta t°$, the equation becomes

$$\Delta H = r\Delta T (t_b - t_s), \tag{110}$$

where r is written in place of the product $e'k'$. This r is a constant which depends only on the nature and area of the radiating

surface, and is called the *radiation constant* of the body. Newton's law is now known to be only a rough approximation to the true law of cooling; but it is simple, and, if the difference in temperature between the body and the surroundings is not greater than 15° or 20°, it holds fairly well.

Let CD and EF in Fig. 22 represent the respective changes in temperature of the body and its surroundings while the body cools by radiation. Since $(t_b - t_s)$ is at any instant the vertical

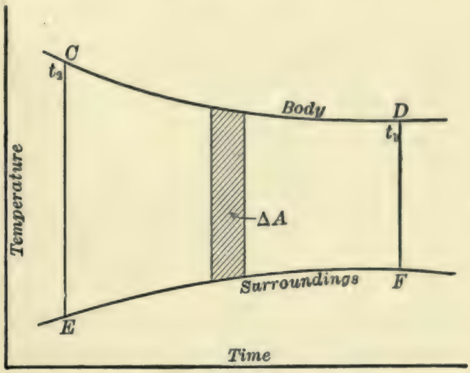

FIG. 22.

distance from EF to CD and ΔT is the horizontal distance between two of these vertical lines which are near together, it follows that the product $\Delta T\ (t_b - t_s)$ is represented very nearly by the shaded area ΔA. That is, from (110),

$$\Delta H = rk\Delta A,$$

where k is a constant which depends upon the scales chosen in plotting. If the temperature of the body, instead of falling the small amount Δt, falls from t_2 to t_1, the entire fall being due to radiation, the total heat lost H involves the sum of the elementary areas ΔA, that is, the area CF. If this area is denoted by A_{CF}, we have

$$H = rkA_{CF}. \tag{111}$$

Now suppose that the body has heat given to it in such a way that its rise of temperature can be represented by the curve

VWX in Fig. 23. The maximum temperature is reached when the body ceases to receive heat from the source faster than it radiates heat to the cooler surroundings. After this point is reached, the body falls in temperature in a manner that can be represented by the line XY. While the body is below the temperature of its surroundings it absorbs heat from them, and while it is above the temperature of its surroundings it loses heat to them. The *radiation correction*, now to be found, is the difference between

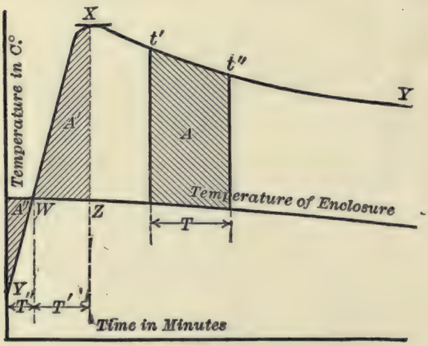

FIG. 23.

the amount of heat lost by the body through radiation and the amount gained by absorption while the body was rising from its original to its maximum temperature.

Let H' denote the heat lost by radiation during the time T' that the body is above the temperature of its surroundings, and H'' the heat gained by absorption during the time T'' that the body is below the temperature of its surroundings. Then from (111) and Fig. 23,

$$H' = rkA' \text{ and } H'' = rkA'',$$

where A' denotes the shaded area WXZ, and A'' the shaded area to the left of VW. Since, in addition, the radiation correction R is, from its definition,

$$R = H' - H'',$$

we have
$$R = rk\,(A' - A''). \tag{112}$$

If the radiation constant r were known, (112) could be used to determine the radiation correction R. The purpose of allowing the body to cool for a time by radiation after the other heat changes have taken place is to make possible the determination of this r. From the part XY of the curve we have from (111), if H denotes the heat lost by radiation while the body falls in temperature from t' to t'',

$$H = rkA,$$

and from (106) and (109) we also have

$$H = e'(t' - t''),$$

where e' denotes the water equivalent of the body. It follows that

$$e'(t' - t'') = rkA. \tag{113}$$

On substituting in (112) the value of r from (113), we have

$$R = e'(t' - t'') \cdot \frac{A' - A''}{A} \cdot \tag{114}$$

(*b*) *Rowland's Method.* — Instead of finding the number of heat units lost by the body due to radiation while the temperature of the body is rising to its maximum value, the effect of radiation can be accounted for if the temperature is determined which the body would have attained if there had been no radiation. In the following modification of a method due to Rowland this temperature can be obtained to a close approximation by a simple graphical construction.

Suppose that a body at a temperature below that of its surroundings is given a quantity of heat H such that its temperature rises to a value above that of the surroundings. While the temperature of the body is lower than that of the surroundings the body absorbs heat, and while the temperature of the body is above that of the surroundings, the body loses heat. The way in which the temperature changes before the heat H is added is represented by the line AB in Fig. 24. The line BD shows how the temperature changes while the body is absorbing the heat H. From B to C the body is, in addition, receiving heat from the surroundings, and from C to D is losing heat to the sur-

roundings. The line DE shows how the temperature of the body changes due to radiation alone.

Through C draw a vertical line. Prolong ED backward until it cuts the vertical line through C in f. Prolong AB forward till it cuts the vertical line at h. Then the temperature change is given by hf.

To see that the above method of finding the desired temperature is reasonable, consider the following. If the heat H had not been given to the body, it would have continued to rise in temperature in the same way that it was rising from A to B, so that by the time

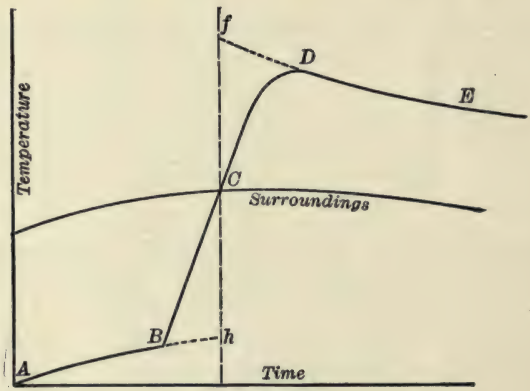

FIG. 24.

it really attained the temperature indicated by C it would have reached the temperature indicated by h. That is, while the body really rose in temperature from B to C, the rise in temperature from B to h was due to heat from the surroundings and the rise from h to C was due to a part of the heat H. Again, if the body had not been given the heat H, but if it had been at first at such a temperature that as it cooled it reached the temperature indicated by D at the same instant that it really reached that temperature — and thereafter cooled as shown by DE — it would have been at a temperature f at the instant when it really was at the temperature C. That is, while the body really rose in temperature from C to D, the fall in temperature due to radiation was the fall from f

to D, so that if there had been no loss of heat by radiation, the rise of temperature during this time would have been from C to f. If, then, there had been no gain nor loss of heat by radiation, the body would have risen in temperature the amount indicated by the distance from h to f.

While the temperature of the body rose from C to D it was really at a lower temperature than if it had been cooling from f to D, and so did not really lose as much heat by radiation as has above been supposed. That is, the point f is higher than it ought to be. For a similar reason h is also somewhat higher than it ought to be. If the time from B to C is about the same as that from C to D, these two errors will nearly balance each other.

(c) Another method should be referred to, although it is considerably less accurate than the two already discussed. In this method, first suggested by Rumford, the initial and final temperatures of the body are so arranged that the difference between the temperature of the surroundings and the initial temperature of the body equals the difference between the temperature of the surroundings and the final temperature of the body. The idea is that, by this arrangement, the heat absorbed from the room while the body is colder than the surroundings equals the heat lost to the room while the temperature of the body is higher than that of the surroundings. That this, however, may be only a rough approximation can be shown as follows:

When a body is heated and then immersed in cold water, the temperature of the water rises in a manner very like that represented by the curve HA in Fig. 25. During the first part of the time, the temperature rises rapidly because the body is at a temperature considerably higher than that of the water, whereas when the temperatures become more nearly the same, the temperature of the water rises more slowly. This means that the first half of the temperature rise is accomplished in less time than the second. And this, in turn, if the temperature of the surroundings is halfway from the lowest to the highest temperature of the water, means that less heat is gained by absorption during the first half of the temperature rise than is lost by radiation during the second half.

In fact, from (111) it follows that in the case represented in Fig. 25 the ratio of the heat lost by radiation to the heat gained by absorption equals the ratio of the areas FAG and HEF. The absorption would compensate the radiation if the temperature of the surroundings were raised to BD, so that the areas CAD and HBC were equal. That is, in the given case, the rise in temperature before reaching the temperature of the surroundings should be about two and a half times that after passing the temperature of the surroundings.

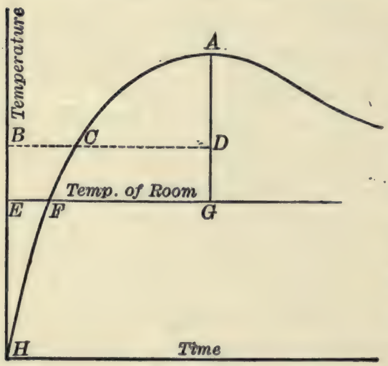

FIG. 25.

31. The Mechanical Equivalent of Heat. — It is found that whenever W units of mechanical energy are entirely used in producing heat, the amount of heat produced is always the same, being independent of the particular way in which the energy is used to produce the heat; that whenever H units of heat are entirely used in producing mechanical energy, the amount of mechanical energy produced is always the same, being independent of the particular way in which heat is used to produce the energy; and that if W units of mechanical energy produce H units of heat, H units of heat produce W units of mechanical energy. These three facts may all be indicated by the one equation

$$W = JH, \tag{115}$$

in which J represents the number of units of mechanical energy that are required to produce one unit of heat. This J is given the name *mechanical equivalent of heat*. Its value depends only upon the units in terms of which the mechanical energy and the heat are measured.

Exp. 65. Determination of the Corrections to be Applied to the Readings of a Mercury-in-Glass Thermometer for (a) Errors due to Displacement of the Fixed Points, (b) Errors due to Exposed Stem

THEORY OF THE EXPERIMENT. — Read Arts. 23 and 27. Due either to mistakes in the original marking of the scale on the stem, or to subsequent volume changes of the glass, the entire scale of a mercury-in-glass thermometer is often displaced in such a way that all readings are in error. If no other cause of error be present, each reading will have the same error. The errors due to this cause will be known if the two "fixed points" are observed.

By definition, the lower fixed point (0° C. or 32° F.) is the temperature of melting ice. The upper fixed point (100° C. or 212° F.) is defined as the temperature of the steam produced by water boiling at sea level and latitude 45° under a barometric pressure of 76 cm. of mercury when the barometer is at the temperature 0° C.

MANIPULATION. — Observe the barometric height, noting the temperature of the barometer by means of the thermometer attached to the instrument. Ascertain from the laboratory instructor the latitude and altitude of the laboratory. From these data compute, in the manner explained in Art. 27, the corrected barometric pressure H reduced to standard conditions.

Suspend the thermometer in the vapor of boiling water. It must not be immersed in the boiling water or be so near the surface that the bulb will be spattered by drops of water, because the temperature of boiling water is not constant but is influenced by the nature of the surface composing the vessel and by the presence of slight quantities of dissolved impurities. The temperature of the vapor, however, depends only upon the pressure. Regnault's hypsometer is very satisfactory for this purpose. It consists of a reservoir R, Fig. 26, in which the water is boiled, surmounted by a tube in which the thermometer is suspended. After passing through this tube the steam passes through the jacket J and escapes into the air at E. For precise work, Guillaume's hypsometer is employed. This consists of the boiler A surmounted by the jacketed tube B in which the thermometer is suspended.

In this instrument, the steam instead of escaping into the air is condensed by a current of cold water circulating in the condenser C, and then trickles back into the boiler.

Both forms of hypsometer have a water manometer M which serves to measure any difference of pressure between the steam inside and the air outside. If the manometer indicates a pressure of d millimeters of water, or $d \div 13.6$ millimeters of mercury, then the total pressure on the surface of the boiling water is $H + (d \div 13.6)$. Call the observed boiling point T_0. Draw the thermometer up until the upper twenty degrees or so of the stem is exposed. After about five minutes note the reading and also the reading at the top of the stopper, draw the thermometer up some twenty degrees farther, and, after about five minutes more, read again at both points. Repeat until the zero point is at the stopper. The difference between the reading at the top of the mercury when the thermometer was wholly immersed and the reading in any of the other cases is the stem exposure correction for that particular case. Note approximately the temperature of the air in the neighborhood of the hypsometer. From (88) calculate the stem exposure correction for each case. Plot on the same sheet two curves, one coördinating the thermometer reading at the stopper and the observed stem exposure correction, and the other coördinating the thermometer reading at the stopper and the calculated stem exposure correction.

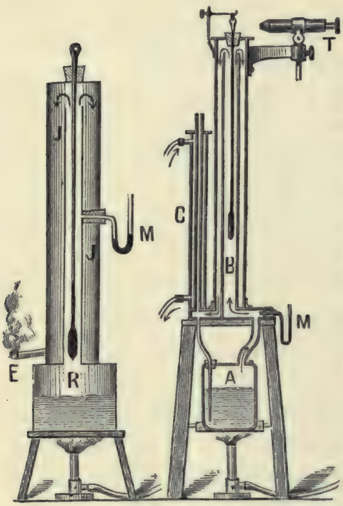

Fig. 26.

Remove the thermometer from the hypsometer, allow it to cool in the air to about 40° C., and then immerse it in a vessel filled with snow or shaved ice which contains enough water to fill the interstices. This gives the depressed zero point.

By reference to Table 3, obtain the temperature of the vapor of water boiling at a pressure of $H + \frac{d}{13.6}$. Call this true temperature T_t. Then $(T_0 - T_t)$ is the error of the upper fixed point, and $(T_t - T_0)$ is the correction to be applied to the reading. Suppose that in the above example the error of the boiling point is found to be $+0°.6$, and the error of the freezing point $+0°.8$. Then the correction for the boiling point is $-0°.6$, and for the freezing point $-0°.8$.

Draw a curve with thermometer readings as abscissæ and corrections as ordinates. This curve shows the error due to displacement of the fixed points.

Exp. 66. Determination of the Corrections to be Applied to a Mercury-in-Glass Thermometer having an Ununiform Bore

Theory of the Experiment. — Read Art. 23. If the bore of a thermometer be not uniform in cross section, the length of the tube corresponding to a degree difference in temperature will not be the same at different parts of the tube. And as it is impossible to get a perfectly uniform capillary, it is necessary to determine the correction to be applied to any particular reading to take account of the irregularity in the bore of a thermometer. The object of this experiment is to construct a curve coordinating thermometer readings and the corrections that must be applied in order to eliminate errors due to irregularity of the bore.

Manipulation. — In the method here to be employed, measurements are made of the lengths of a short thread of mercury when in different parts of the tube, and from these lengths points are found throughout the whole length of the tube that separate equal volumes. The length of the thread to be broken off depends upon the thermometer. If the thread is too long, local irregularities of bore are not evident; if the thread is too short, its changes in length are minute. If a dividing engine is available, a thread not more than a centimeter long is advisable. If no magnification is to be used and the thermometer is an ordinary Centigrade ther-

mometer graduated from 0° to 100° in degrees, a thread some fifteen degrees long is perhaps most satisfactory.

The separation of the calibrating thread requires some dexterity. In blowing the bulb on a thermometer tube, a slight constriction is usually left where the bulb and tube join. If such a thermometer is inverted and then given a sudden jar, the thread is likely to separate at this point. If there be no such constriction, the thread may be separated by laying the thermometer on a table and striking the upper end of the tube with a small block of wood. If this is not carefully done, however, cracks may be produced inside the stem near the bulb. If the bore has an enlargement at the upper end, the column of mercury that has been broken off is allowed to run into this enlargement and to remain there while the tube is being calibrated. The bulb is then slightly warmed until a thread of mercury of the proper length runs into the tube, and this, in turn, is separated from the mercury in the bulb. This is the thread that is used in the calibration. If the capillary has no enlargement at the upper end in which to store part of the mercury, it may be necessary to use two mercury threads to calibrate the two ends of the tube. When this is the case, the bulb is cooled, with a mixture of ice and salt if necessary, until all the mercury has run into the bulb except the length that is to be broken off. This thread is separated and run to the farther end of the tube. In order to make measurements in the lower end of the tube, this part of the thermometer must be freed of mercury and another thread separated as before.

When a thread has been broken off, it is to be brought nearly to one end of the tube and the position of both ends carefully read, then moved along through a quarter or a third of its length and the positions of both ends again read, this process being repeated until the thread has been moved to the other end of the tube. Suppose that when this is done — a mirror being used, and readings being made to twentieths of a degree — the readings are those in the first, second, fourth, and fifth columns of the following table:

Lower end of thread at	Upper end of thread at	Thread length in degrees	Lower end of thread at	Upper end of thread at	Thread length in degrees
−12°.30	+ 3°.60	15°.90	39°.95	55°.65	15°.70
− 7 .25	8 .65	15 .90	45 .30	60 .95	15 .65
− 2 .05	13 .85	15 .90	50 .00	65 .65	15 .65
+ 3 .00	18 .85	15 .85	55 .20	70 .80	15 .60
8 .15	24 .00	15 .85	60 .15	75 .75	15 .60
14 .30	30 .15	15 .85	64 .90	80 .45	15 .55
20 .00	35 .80	15 .80	70 .10	85 .60	15 .50
24 .55	40 .35	15 .80	74 .85	90 .35	15 .50
30 .05	45 .80	15 .75	80 .05	95 .50	15 .45
34 .90	50 .65	15 .75	83 .30	98 .75	15 .45

The quantities in the third and sixth columns are calculated from the observed quantities. The quantities in the first and third columns give the following curve, Fig. 27, which shows the

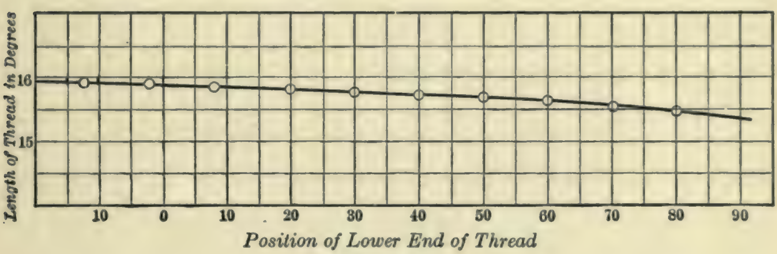

FIG. 27.

length of the thread when at different points of the capillary. From this curve the lengths of equal volume portions of the tube can be determined as follows:

The curve shows that if the bottom of the mercury thread were at 0° its length would be 15°.87, so that the top of the thread would be at 15°.87; it also shows that if the bottom of the thread were at 15°.87 its length would be 15°.81, so that the top of it would be at 31°.68; if the bottom of the thread were at 31°.68 its length would be 15°.73; etc. These values are recorded in the following table:

Points on scale between which volumes of bore are equal	Lengths of thread between equal volume points	Positions of equal volume points if bore had been uniform	Corrections for points in first column
0.00	15.87	0.00	±0.00
15.87	15.81	15.69	−0.18
31.68	15.73	31.37	−0.31
47.41	15.66	47.06	−0.35
63.07	15.57	62.75	−0.32
78.64	15.48	78.44	−0.20
94.12	Av. 5.687	94.12	±0.00

The quantities in the third column are found by multiplying by 1, 2, 3, etc., the average length of thread between equal volume points. The quantities in the fourth column are found by subtracting the quantities in the first column from those in the third. The quantities in the first and fourth columns give the upper curve in Fig. 28. This curve gives the corrections that must be applied

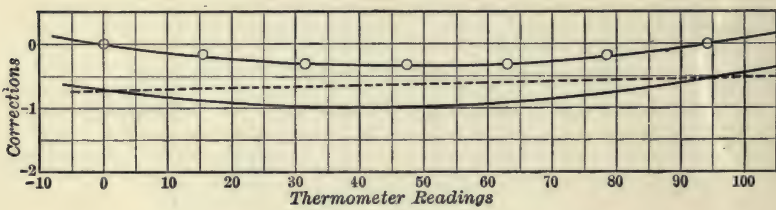

Fig. 28.

to readings at different points along the scale on account of irregularities in the bore.

If now on the same sheet of coördinate paper on which the correction curve for irregularities of bore was plotted, the freezing point correction be entered along the axis of ordinates opposite the zero of abscissæ, and the boiling point correction be entered opposite the observed boiling point, and these points be connected by a straight line, as shown by the dotted line in Fig. 28, this line gives the corrections for all intermediate points of the scale due to the displacement of the fixed points.

By adding the ordinates of the correction curve for the irregularities of bore — the upper curve — to the corresponding ordinates of this correction curve for displacement of the fixed points — the dotted line — the lower curve in Fig. 28 is obtained. This is called the Calibration Curve of the thermometer.

If this calibration be done with two mercury threads instead of one, the calibration should extend from each end to a distance past the middle of the tube. The curve analogous to that in Fig. 27 will be a continuous line, but along the region where data were taken with both mercury threads, one branch of the curve will be above the other. In this region find the ratio of the ordinates of the two curves for three or four positions on the thermometer scale. This ratio must be really the same for all points on the thermometer scale. By multiplying any ordinate of one curve by the averages of the values found for the ratio, the corresponding ordinate of the other curve will be obtained. Proceeding in this manner, a continuous curve is obtained, just as though all of the calibration had been performed with a single mercury thread.

Exp. 67. The Flash Test, Fire Test, and Cold Test of an Oil

THEORY OF THE EXPERIMENT. — If an inflammable gas is mixed with air in proper proportion, the mixture will explode on ignition. The air above a volatile oil is saturated with the oil vapor. If the temperature of the oil is slowly raised, the proportion of oil vapor in the air will increase until, at a certain temperature, the saturated air will become an explosive mixture. This temperature is called the *flash point* of the oil. If the temperature of the oil is still farther increased, a point will be reached at which the oil will evolve vapor so rapidly that, when ignited, it will burn continuously. This is called the *fire test* of the oil. The *cold test* of an oil is the lowest temperature at which the oil will flow. The object of this experiment is to make a flash test, fire test, and cold test of a sample of oil.

The general method of determining the flash point is to heat the specimen gradually in a covered cup and at frequent intervals

pass a small flame near the surface of the oil. In making a fire test, the specimen is heated in an open cup and the temperature is noted at which the vapor will burn continuously when ignited. The flash point depends upon (*a*) the rate of heating, (*b*) the depth and diameter of the cup, (*c*) whether the cup is closed or open, (*d*) the quantity of oil used, (*e*) the size of the testing flame and its distance from the surface of the oil. Consequently, the size and design of the testing apparatus and the method of carrying out a determination are explicitly described in the legislative enactments of the various states.

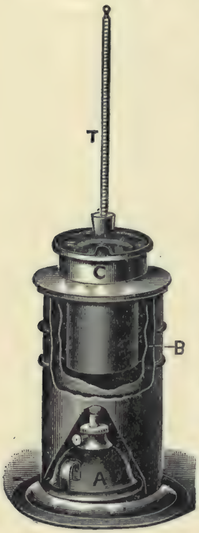

FIG. 29.

MANIPULATION. — The form of apparatus most commonly used in this country for the flash point is the "New York State Board of Health Tester." This consists, Fig. 29, of a seamless copper cup *C* covered by a glass plate perforated with two holes — one for the insertion of the thermometer and another for the testing flame. This cup is heated in a water or air bath *B* by means of an alcohol lamp or small Bunsen burner. The whole apparatus should be placed in a sheet-iron pan filled with sand.

In using this apparatus to test illuminating oils, the New York State Board of Health publishes * the following regulations:

"Remove the oil cup and fill the water bath with cold water up to the mark on the inside. Replace the oil cup and pour in enough oil to fill it to within one-eighth of an inch of the flange joining the cup and the vapor chamber above. Care must be taken that the oil does not flow over the flange. Remove all air bubbles with a piece of dry paper. Place the glass cover on the oil cup, and so adjust the thermometer that its bulb shall be just covered with oil.

"If an alcohol lamp be employed for heating the water bath, the wick should be carefully trimmed and adjusted to a small flame. A small Bunsen burner may be used in place of the lamp.

* Report of N. Y. State Board of Health, 1882.

The rate of heating should be about two degrees per minute, and in no case exceed three degrees.*

"As a flash torch, a small gas jet one-quarter of an inch in length should be employed. When gas is not at hand, employ a piece of waxed linen twine. The flame in this case, however, should be small.

"When the temperature of the oil has reached 85° F., the testing should commence. To this end insert the torch into the opening in the cover, passing it in at such an angle as to well clear the cover, and to a distance about halfway between the oil and the cover. The motion should be steady and uniform, rapid and without a pause. This should be repeated at every two degrees' rise of the thermometer until the thermometer has reached 95°, when the lamp should be removed and the testings should be made for each degree of temperature until 100° is reached. After this the lamp may be replaced if necessary and the testings continued for each two degrees.

"The appearance of a slight bluish flame shows that the flashing point has been reached.

"In every case note the temperature of the oil before introducing the torch. The flame of the torch must not come in contact with the oil.

"The water bath should be filled with cold water for each separate test, and the oil from a previous test carefully wiped from the oil cup."

Make five determinations of the flash point and take the mean. After each determination, remove the cover from the oil cup and blow the burnt gases out of the cup.

After the flash point has been determined, remove the cover from the oil cup and continue to heat the oil at the rate of two degrees per minute. About every half minute test the oil with the small flame as above described. The lowest temperature at which the vapor of oil will burn continuously is the fire test. Remove the thermometer and smother the flame by placing on top of the oil cup a piece of asbestos board. Such a damper should always be at hand for emergencies.

* This refers to degrees Fahrenheit.

In the case of lubricating oils the method of finding the flash point and the fire test is exactly as above described except that the rate of heating should be 15° F. per minute and the testing flame should be applied first when the oil is about 200° F.

In making the cold test, a glass vial or boiling tube of about 100 cc. capacity is one-fourth filled with the oil under investigation, and then placed in a freezing mixture of ice and salt. When all of the oil has congealed, it is removed from the freezing mixture and thoroughly stirred with a thermometer until it is sufficiently softened to flow from one end of the tube to the other. The temperature at which this occurs is the cold test of the oil.

Exp. 68. Relation between Boiling Point and Concentration of a Solution

THEORY OF THE EXPERIMENT. — Read Art. 27. The object of this experiment is to find the relation between the boiling point and the concentration of a solution of common salt.

The boiling point of a solution of a non-volatile substance is higher than the boiling point of the pure solvent. If a current of steam be passed into an aqueous solution below its boiling point, steam will be condensed in the solution until the heat thereby liberated raises the temperature of the solution to its boiling point. Consequently steam that passes through a solution will leave it at the boiling point of the solution and not at that of pure water. However, as the steam escapes into the space above the solution it cools somewhat by expansion, and wherever it comes into contact with the walls of the vessel, with the thermometer, or with any body that can gradually conduct away the heat given up by the condensation of the steam, this cooling continues until the steam becomes saturated, that is, until temperature falls to the boiling points of pure water. Consequently, in determining the boiling point of the pure solvent the thermometer is suspended in the space above the liquid, while in determining the boiling point of a solution the thermometer bulb must be immersed in the solution.

Buchanan has recently utilized the principle stated in the

preceding paragraph for finding the boiling point of a saturated solution. A quantity of the pure solute is placed in the bottom of a tall test tube containing a thermometer. A current of steam is sent through a glass tube extending to the bottom of the test tube until a saturated aqueous solution of the given solute is obtained. As long as any of the solute remains undissolved and the current of steam is uninterrupted, the temperature of this saturated solution remains at its boiling point.

MANIPULATION. — The apparatus used in determining the boiling point of a dilute solution consists of a flask provided with a cork fitted with a thermometer and condenser. Without the condenser the solution would gradually increase in concentration through the loss of steam. To prevent "bumping," a handful of clean dry pebbles or pieces of broken glass is placed in the flask. With the flask about one-third filled with a solution of some 50 g. of sodium chlorid to each liter of water, insert the thermometer until its bulb is 5 cm. or more above the surface of the liquid. With the condenser in place, heat the solution until it boils fairly rapidly. Read the thermometer and also the laboratory barometer. Push the thermometer down until its bulb is in the boiling solution, and when it has become steady read it again.

FIG. 30.

From the corrected barometer reading (see Art. 27 and Table 3) the true temperature of the vapor of boiling water can be found. This value minus the reading of the thermometer when in the vapor is the correction to be applied to the given thermometer in the neighborhood of 100°. This correction added to the reading of the thermometer when in the boiling solution gives the boiling point of this solution. In the same manner find the boiling points of solutions which contain respectively three and five times as large a proportion of salt.

To find the boiling point of a saturated solution, fasten a large boiling tube in a vertical position in a retort stand, and

fill the tube to a depth of one or two centimeters with salt. Suspend in the tube the same thermometer used before so that the bulb just touches the layer of salt and then push halfway through the layer of salt the end of a glass tube in which flows a current of steam. When the bulb of the thermometer is submerged in the solution formed by the salt and condensed steam, observe the temperature. When the correction determined in the first part of the experiment is applied to this reading, it gives the required boiling point.

Plot and find the equation of the curve showing the relation between concentration and corrected boiling point. The concentrations may be expressed in grams of salt per liter of water, and the concentration of a saturated solution of sodium chlorid may be taken as 396.5 g. per liter. The temperature of the vapor above the boiling solutions is the boiling point of a solution of zero concentration.

Exp. 69. Determination of the Coefficient of Linear Expansion of a Solid

THEORY OF THE EXPERIMENT. — Read Art. 25. The object of this experiment is to determine the coefficient of linear expansion of a metal. If l_1 denotes the length of the body at temperature t_1 and l_2 its length at temperature t_2, we have from (90),

$$l_1 = l_0 (1 + \alpha t_1) \tag{116}$$

and

$$l_2 = l_0 (1 + \alpha t_2). \tag{117}$$

On dividing each member of (117) by the corresponding member of (116) we obtain

$$\frac{l_2}{l_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1}.$$

Whence, solving for α,

$$\alpha = \frac{l_2 - l_1}{l_1 t_2 - l_2 t_1}. \tag{118}$$

If the specimen being studied be in the form of a long wire or rod, it may be suspended vertically, surrounded by a steam jacket, and its change of length obtained by means of an optical

lever. If the specimen be in the form of a short rod or tube, the change of length $(l_2 - l_1)$ can be conveniently found by the device illustrated in Fig. 31. The specimen, enclosed in a steam jacket, is held horizontally in two wyes, one of which, M, is fixed, while the other, N, is fastened to a horizontal plate of glass resting on two rollers made of hardened steel rods. This movable support with its two rollers resting on a glass bedplate constitutes a carriage which moves when the length of the specimen changes. A light pointer fixed to one of the rollers moves over the face of a divided circle. If the roller carrying the pointer be situated directly below the wye supporting the movable end of the specimen, the indication of the pointer will be unaffected by any change in the temperature of the carriage.

When the rod is heated, the carriage is pushed forward a distance $(l_2 - l_1)$ and the pointer is turned through an angle θ. During this motion the carriage has advanced a certain distance with respect to the roller, and the bedplate has moved backward an equal distance with respect to the roller. That is, with respect

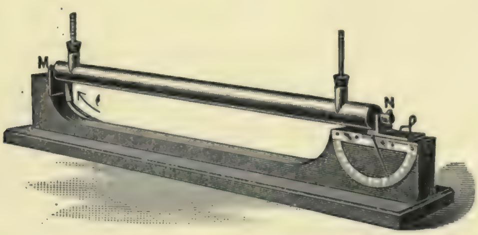

FIG. 31.

to the bedplate the roller has moved forward half as far as has the carriage, *i.e.*, the roller has moved a distance $\frac{1}{2}(l_2 - l_1)$. If the diameter of the roller is called d, then the distance that the roller has moved on the bedplate is also $\frac{\theta}{360} \cdot \pi d$. Whence,

$$\tfrac{1}{2}(l_2 - l_1) = \frac{\theta}{360} \cdot \pi d,$$

and, therefore, from (118),

$$\alpha = \frac{\theta \pi d}{180\,(l_1 t_2 - l_2 t_1)}. \tag{119}$$

Manipulation. — Measure the diameter of the roller with a micrometer caliper. Assemble the apparatus, being careful that the roller carrying the pointer is normal to the length of the bed-plate, and also that it is at the center of the divided circle. It is well to start with the pointer about as far to one side of the vertical as it will come to be on the other side of the vertical. It can be set in this way after a preliminary experiment in which the angle through which it will turn is determined roughly. The carriage should be so placed that the wye is directly above the roller that carries the pointer. Measure l_1, the distance between the edges of the two wyes on which the specimen rests, and note the readings of both thermometers and of the pointer. Send a current of steam for some minutes through the jacket surrounding the specimen and then observe the new position of the pointer and again read both thermometers. By (88) correct the thermometer readings for steam exposure. Calculate α by (119).

Exp. 70. Determination of the Absolute Coefficient of Expansion of a Liquid by the Method of Balancing Columns

Theory of the Experiment. — Read Arts. 25 and 27. The object of this experiment is to determine the coefficient of expansion of a liquid by a method which is independent of the change in volume of the containing vessel. The method employed in this experiment is to determine the coefficient of expansion of the liquid from the ratio of its densities at different temperatures.

The apparatus used by Regnault is illustrated in Figs. 32 and 33. Consider a W-shaped tube $ABCDEF$ containing the liquid, having the branch A kept at a high temperature by means of a steam jacket, and the remainder of the apparatus at the temperature of the tap water by means of water jackets. The liquid columns A and F are connected at the top by the tube G, so that the pressures of the liquid in both columns are the same at this level. At

the bottom the two columns A and F are kept separated by means of compressed air in the tube CD. Let H_1, H_2, h_1, and h_2 represent the differences in level indicated in the figure. Let the temperature and density of the liquid in the hot part of the apparatus be denoted by t_2 and ρ_2 respectively, and the temperature and density

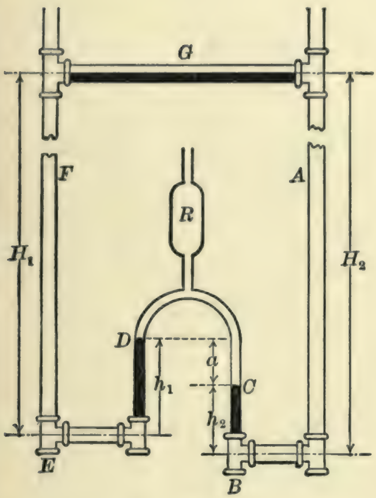

FIG. 32.

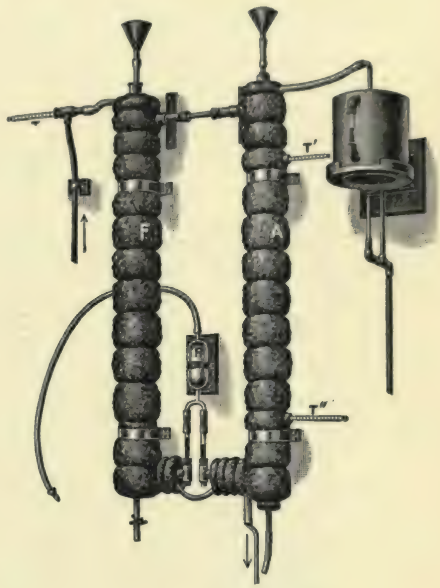

FIG. 33.

of the remainder of the liquid by t_1 and ρ_1. Let P denote the atmospheric pressure and P' the pressure of the air in R. Then the pressure at the bottom of the F-column is $(P + \rho_1 g H_1)$, and the pressure at the bottom of the D-column is $(P' + \rho_1 g h_1)$. Now these are pressures at the same level in a fluid at rest and are therefore equal. That is,

$$P + \rho_1 g H_1 = P' + \rho_1 g h_1. \tag{120}$$

In the same way for the A-column and the C-column,

$$P + \rho_2 g H_2 = P' + \rho_1 g h_2. \tag{121}$$

On subtracting each member of (121) from the corresponding member of (120) and solving for $\frac{\rho_1}{\rho_2}$,

$$\frac{\rho_1}{\rho_2} = \frac{H_2}{H_1 - h_1 + h_2}. \tag{122}$$

From the figure, $H_1 = H_2 - b$

and $h_1 + b = h_2 + a, \quad \text{or,} \quad -h_1 + h_2 = b - a.$

Whence (122) may be written

$$\frac{\rho_1}{\rho_2} = \frac{H_2}{H_2 - a}. \tag{123}$$

Now the density of a given mass is inversely proportional to its volume. So that, if β denotes the coefficient of expansion of the liquid,

$$\frac{\rho_0}{\rho_1} = \frac{v_1}{v_0} = 1 + \beta t_1 \tag{124}$$

and

$$\frac{\rho_0}{\rho_2} = \frac{v_2}{v_0} = 1 + \beta t_2. \tag{125}$$

On dividing each member of (125) by the corresponding member of (124), a value is obtained for $\frac{\rho_1}{\rho_2}$. If this value be equated to that in (123), and the resulting equation solved for β, we obtain

$$\beta = \frac{a}{H_2 (t_2 - t_1) - a t_2}. \tag{126}$$

Manipulation. — An instructor will pump air into the reservoir R so that the liquid rises in the tubes A and F until its surface stands about at the axis of the tube G. Water should flow slowly through the water jacket. The temperature t_1 is to be taken by a thermometer, and the temperature of the hot jacket calculated from a reading of the barometer.

Measure H_2 with a meter stick and a with a cathetometer. Make at least five independent determinations of a, readjusting the cathetometer before each one, and use the mean.

Exp. 71. Determination of the Coefficient of Expansion of a Gas by Means of an Air Thermometer

THEORY OF EXPERIMENT. — Read Arts. 26 and 27. If a given mass of any substance be heated from 0° to 1°, and the pressure upon it kept constant, the ratio of the increase of volume to the initial volume is called the *coefficient of expansion* of the substance. If it be heated from 0° to 1°, and its volume kept constant, the ratio of the increase of pressure to the initial pressure is called the *temperature coefficient of pressure* of the substance. In Art. 26 it is shown that for a perfect gas these two quantities are equal. Since it is easier to measure the pressure of a gas under constant volume than to measure the volume under constant pressure, the coefficient of expansion will be determined in the present experiment from observations of the changes produced in the pressure of a gas when its mass and volume remain nearly constant and its temperature changes.

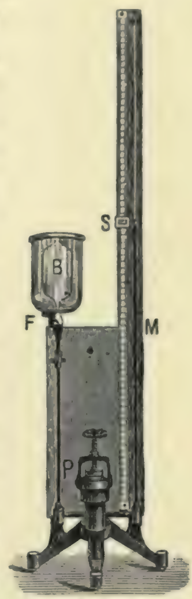

FIG. 34.

An apparatus well suited for determining the pressure coefficient is some form of Jolly's Air Thermometer. This consists (Fig. 34) of a glass bulb B filled with air or other gas, connected to an open manometer tube M filled with mercury. Immediately below the bulb is a tube containing an index finger F made of colored enamel. The volume of the gas is made definite by adjusting the plunger P until the mercury surface is brought to the point F. The pressure of the gas in the bulb is measured by the difference between the levels of the mercury surface at F and the mercury surface in the manometer tube. The bulb is inclosed by a vessel in which can be placed water or ice. This vessel is provided with a close fitting cover with a hole for the passage of a steam delivery tube. D is a drying tube used in filling the bulb.

On account of the temperature of the small amount of gas in the exposed part of the bulb being different from that in the jacketed part of the bulb, and also on account of the change in the volume of the bulb when its temperature is changed, (97) would need to be modified when results of great precision are required. In the present experiment, however, these refinements will not be made.

MANIPULATION. — The student will find the bulb filled with dry air ready for use. Fill the jacket with water at about the temperature of the room. While the air in the bulb is attaining the temperature of the water, read on the manometer scale the height of F. This can be done with sufficient precision by means of a straight edge and spirit level. Stir the water and observe its temperature. Adjust the plunger P until mercury just touches the index F. Set the slider S tangent to the meniscus in M and note its position on the scale.

Siphon the water out of the jacket. Replace the cover and pass steam into the jacket. By adjusting the plunger, keep the meniscus in contact with the index F. When no farther adjustment seems necessary, set the slider and note its position. Read the laboratory barometer. The temperature of the air in the bulb during this operation is obtained by correcting the barometric reading to standard conditions and referring to Table 3.

Now fill the jacket with shaved ice and adjust the plunger as before. Be certain that the air in the bulb is at the temperature of the melting ice before noting the position of the slider on the scale.

We are now in position to compute the coefficient of expansion of air from 0° to the temperature t of the room, and also from 0° to 100°.

Exp. 72. Determination of the Maximum Vapor Pressure of a Liquid at Temperatures below 100° C. by the Static Method

THEORY OF THE EXPERIMENT. — When a liquid evaporates in a closed space, the vapor formed produces on the surface of the liquid and on the inclosing walls a pressure which increases with the mass of vapor and with the temperature. For a given tem-

perature the vapor pressure * reaches a maximum value when the space is saturated. The object of this experiment is to determine the pressure of saturated aqueous vapor at temperatures from about 50° C. to 100° C.

In the method to be used in this experiment, the vapor pressure is determined from the decrease in the height of a barometer column produced by a small quantity of the specimen which is introduced into the vacuous space above the mercury. The apparatus (Fig. 35) consists of a barometer tube, the upper end of which is enlarged into a narrow bulb and its lower end joined to an open manometer tube, *M*. Opening into the horizontal tube joining the barometer and manometer is an iron cylinder filled with mercury. The height of mercury in the two tubes can be varied by means of a plunger, *P*, in this cylinder. A small enamel finger, *F*, in the bulb of the barometer tube serves as a convenient fixed point from which heights can be measured. The vapor being studied can be brought to the desired temperature by means of a water bath surrounding the bulb.

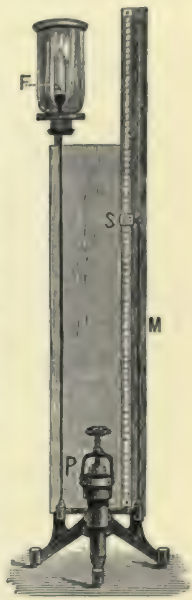

Fig. 35.

The pressure in the tube *M* at the level of the mercury surface at *S* is the atmospheric pressure. The pressure of the vapor in the bulb is less than this by the pressure due to the column of liquid between the levels of *S* and *F*.

Manipulation. — The bulb has been freed from air, a specimen of air-free water introduced, and the upper end of the bulb permanently sealed.

Observe the atmospheric pressure from the laboratory standard barometer. Fill the water jacket with water and pass steam into it until it reaches a temperature of about 25° C. On account of

* The expression *vapor tension* is sometimes used instead of *vapor pressure* to denote elastic stress exerted by a vapor. Careful writers, however, use the word *pressure* to denote a push, and *tension* to denote a pull. Since vapors and gases cannot exert a pull, the term *vapor tension* is a misnomer.

danger of cracking the glass, the current of steam should not be directed against the bulb nor against its jacket. Hold the temperature as nearly steady as possible for a few minutes, and by means of the plunger adjust the height of the mercury in the barometer tube until it is brought just into contact with the tip of the index finger. Stir the water in the jacket, observe its temperature, readjust the plunger if necessary, move the slider S until its index line is tangent to the meniscus in the manometer tube, and read the position of this index line. Determine to the nearest millimeter the height of the water column above the mercury. Divide this height by 13.6, the specific gravity of mercury, and add the result to the difference between the levels of the mercury in the two tubes. Subtract this result from the barometric pressure as given by the standard barometer. The result is the vapor pressure of water at the temperature of the experiment.

Take similar readings every ten degrees up to about 95° C. When through the experiment, draw off the water in the jacket, and adjust the mercury to about the same level in both arms. Plot a curve with vapor pressures as ordinates and corresponding temperatures as abscissæ. On the same coördinate axes plot another curve from the values given in Table 4.

This method is liable to several errors. The surface tension of the dry mercury in the manometer tube is different from that of the wet mercury in the barometer tube. This will cause a rise of the column having the wet surface of 0.10 to 0.15 mm. The fact that the lower part of the barometer tube is at a lower temperature than the upper causes the final result to be too low. This error will be of the order of 0.15 mm. If the position of the end of the index finger is read through the water jacket, the refraction of the glass and water will introduce an uncertainty that may amount to 0.5 mm. This error is obviated by carefully measuring the distance from the end of the index to a fine scratch on the tube below the water jacket before the apparatus is assembled. In order to use this scratch as the fiducial line from which heights are measured, the position of the line is read on the meter stick by means of a cathetometer. The greatest limitation to the

use of this method, however, is due to the large error introduced in the depression of the barometer column by an impurity of the specimen.

Exp. 73. Determination of the Maximum Vapor Pressure of a Liquid at Various Temperatures by the Dynamic Method

THEORY OF THE EXPERIMENT. — Read Art. 27. The object of this experiment is to determine the maximum vapor pressure of water at various temperatures from about 50° C. to about 120° C. The dynamic method to be employed in this experiment is based upon the following two laws of vapors: first, a liquid boils when the pressure of its vapor equals the external pressure; second, if the pressure does not change, the temperature of the boiling liquid remains constant as long as there is any liquid to vaporize.

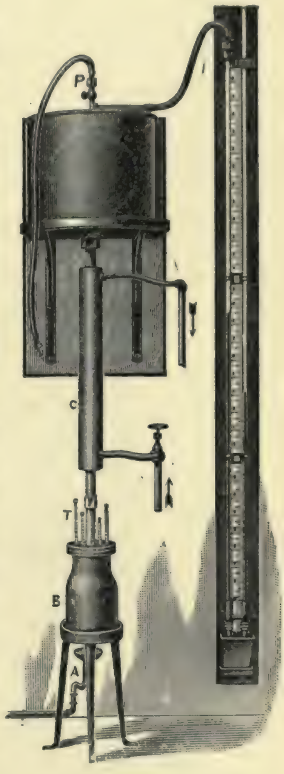

FIG. 36.

In Regnault's apparatus (Fig. 36) the water is inclosed in a boiler *B*, from the top of which runs a tube through the condenser *C* to a large metal reservoir inclosed in a water bath kept at constant temperature. The reservoir is filled with air the pressure of which is varied by means of a pump connected to *P*. The air in the reservoir serves to equalize any sudden changes of pressure due to irregularities in boiling. If it were not for the condenser, most of the steam formed in the boiler would not be condensed, but instead would increase the pressure in the boiler and reservoir and thus change the boiling point. That is, when the burner was lighted both temperature and pressure would

gradually rise. The pressure of the vapor is measured by means of the open manometer at the right. The temperature of the vapor in the boiler is obtained from the four thermometers T, placed in tubulures which project into the boiler. The bottoms of all of them are filled with mercury, so that the bulbs of the thermometers quickly acquire the temperature of the vapor in the boiler.

MANIPULATION. — The boiler already contains sufficient water. Start a stream of cold water flowing through the condenser, and then light the gas at the burner under the boiler. Pump air out of the reservoir until the pressure is reduced to about 10 cm. of mercury, *i.e.*, until the difference between the heights of the mercury in the two arms of the manometer is about 65 cm. Then close the stopcock in the tube connecting the pump and the reservoir. When the thermometers have become steady, record the reading of each thermometer and of each of the manometer tubes. Note also the temperature of the manometer and the barometric height. The corrected barometric height diminished by the corrected difference of level between the manometer columns gives the pressure of the vapor at the temperature indicated by the thermometers in the boiler. Assuming that the manometer scale is correct at 20° C., reduce the pressure to 0° C., *i.e.*, so correct it as to make it the pressure that would have been observed if barometer and manometer had been at 0° C. This can be effected for the barometer by (104) and for the manometer by (102).

Allow air to enter the reservoir until the difference in the heights of the mercury columns is about 20 cm. less than before. This increase of pressure requires that a higher temperature be attained before the water will boil. When the temperature has reached the new boiling point, a second set of observations is to be made. In the same manner, the boiling points corresponding to about eight different pressures are to be determined, the difference in pressure in passing from each case to the next being about 20 cm. of mercury.

Plot a curve with pressures as ordinates, and temperatures as abscissæ. On the same coördinate axes plot for the same range

another curve with values from Table 4. This curve, showing the way in which the pressure of saturated water vapor changes with temperature, is called the *steam line.*

Exp. 74. Determination of the Water Equivalent of a Calorimeter

THEORY OF THE EXPERIMENT. — Read Arts. 28 and 29. In many measurements of heat quantity the water equivalent of the apparatus must be known. Any apparatus used to measure quantities of heat is called a *calorimeter*. The simplest calorimeter consists of a thin copper vessel held centrally within a jacket, Fig. 37. The inner vessel contains a thermometer T' and stirrer S while a second thermometer T is suspended between the inner and outer vessels. The water equivalent of such a calorimeter is the number of grams of water which require the same amount of heat to cause a given change in temperature that is required to produce an equal change in the temperature of the inner vessel with the stirrer and thermometer.

If the mass and specific heat be known for each part of the calorimeter that changes in temperature, the water equivalent is most easily and most accurately obtained by taking the sum of the products of these masses and the corresponding specific heats (109).

When this method cannot be applied, the water equivalent may be determined experimentally by the Method of Mixtures. The principle of this method is that if two or more bodies at different temperatures be placed in thermal contact in such a manner that all heat changes take place exclusively between these bodies, the quantity of heat lost by one body equals that gained by the others. When employing the method of mixtures, first weigh the inner vessel and stirrer, then fill about one-half full of cold water at temperature t_c, and weigh again. The difference of weight gives the mass m_c of cold water. On adding m_h grams of hot water at temperature t_h, the temperature of the mixture will have some value t_m. Since the heat lost by the hot water equals that gained by the calorimeter and contents, plus that lost to the surroundings,

$$m_h c_w (t_h - t_m) = m_c c_w (t_m - t_c) + e c_w (t_m - t_c) + R',$$

where e represents the water equivalent of the calorimeter, c_w represents the thermal capacity of water, and R' the radiation correction. By selecting proper values for the temperatures of the hot water and cold water, R' may be made very small (Art. 30c). When R' is so small that it may be neglected, we have from the above equation,

$$e = \frac{m_h(t_h - t_m)}{t_m - t_c} - m_c. \tag{127}$$

The satisfactory determination of a water equivalent by this method requires deft and rapid manipulation and careful determination of temperature.

MANIPULATION. — Weigh the inner vessel with the stirrer. Half fill the inner vessel with water at a temperature some 10° or 15° below that of the room and weigh again. The difference is the mass of cold water, m_c. Prepare another vessel of water some 15° or 20° above the temperature of the room and note simultaneously the temperature t_c of the cold water and the temperature t_h of the hot water. Then quickly pour into the inner vessel enough of the hot water nearly to fill it. Meantime stir briskly and watch the thermometer in the calorimeter. After noting the temperature of the mixture t_m, weigh again to determine the mass m_h of hot water added. These values substituted in (127) give the water equivalent required.

Exp. 75. Determination of the Specific Heat of a Solid by the Method of Mixtures

THEORY OF THE EXPERIMENT. — Read Arts. 28, 29, and 30b. The Method of Mixtures depends upon the principle that when a number of bodies of different temperatures are brought together, the amount of heat lost by the bodies that fall in temperature equals the amount of heat gained by the bodies that rise in temperature.

Consider a body of mass m, thermal capacity c, and temperature t, to be placed in a mass m_l of liquid of thermal capacity c_l and temperature t_l contained in a vessel of mass m_c made of a material whose thermal capacity is c_c. Let the final temperature of the mix-

ture be t_f. Then, if t is higher than t_l, the heat lost by the given body equals the sum of the heat gained by the vessel and its contents and that gained by the surrounding air. That is, from (106), if R denotes the radiation correction,

$$mc\,(t - t_f) = (m_l c_l + m_c c_c)\,(t_f - t_l) + R. \qquad (128)$$

The correction for radiation may be applied either by Regnault's method of calculating R, or graphically by the modification of Rowland's method given in Art. 30. When the specimen is in small pieces the rise of temperature is rapid and Rowland's method is perhaps to be preferred.

If water is the liquid used, $c_l = 1$. For purposes of abbreviation the water equivalent of the vessel, $m_c s_c$, will be denoted by the single letter e. Then if $(t_f' - t_l')$ denotes the temperature change of the mixture which would have occurred if there had been no gain or loss by radiation, (128) gives

$$\frac{c}{c_l} = s = \frac{(m_l + e)\,(t_f' - t_l')}{m\,(t - t_f')}, \qquad (129)$$

where s is the specific heat of the specimen.

It should be noted that e represents the water equivalent of the vessel in which the mixing occurs, together with any accessories it may contain, such as a stirrer or thermometer.

MANIPULATION. — The special apparatus used in this experiment consists of a calorimeter and a heater. A calorimeter is any apparatus used to measure quantities of heat. The ordinary "water calorimeter" used in this experiment (Fig. 37) consists of a thin polished copper vessel held centrally within a jacket by means of non-conducting supports. The inner vessel contains a thermometer T' and stirrer S, while a second thermometer T is suspended in the air space, between the two concentric vessels. A convenient form of heater, shown in Fig. 38, consists of a closed copper can in which water can be boiled. Extending through one side and projecting nearly through the boiler at an angle of 45° with the bottom is a tube sealed at the lower end and having the upper end closed with a cork through which extends a thermometer. The specimen to be heated is placed in this tube, and when the

temperature indicated by the thermometer *T* has become steady, the specimen is dropped into the calorimeter. If the specimen is in small pieces, *e.g.*, lead shot, it can be poured into the calorimeter by simply tilting the heater; if the specimen is in a single piece, it

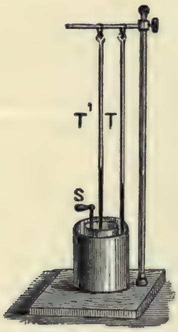

FIG. 37.

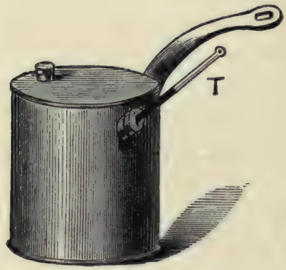

FIG. 38.

is drawn out of the heater with a thread and quickly lowered into the calorimeter.

In finding the specific heat of lead shot, fill the part of the heating tube that is surrounded by steam with the specimen. While the specimen is heating to a constant temperature, weigh the inner vessel of the calorimeter, fill about two-thirds full of water at a temperature three or four degrees below that of the room and reweigh. The difference gives the mass of water in the calorimeter. Assemble the parts of the calorimeter, placing one thermometer in the water contained in the inner vessel and another thermometer against the inner surface of the jacket. The thermometer in the water should have its bulb entirely covered by the water, but should not be low enough to be touched by the specimen. The temperature of the water should now be observed at half-minute intervals, and the temperature of the jacket every minute or two. For each reading, the hour, minute, and second at which the reading is made should be recorded. The readings are taken continuously but belong to three successive periods.

Before beginning the first period be sure that the thermometer

in the heater is steady in the neighborhood of 100° and record its reading.

First period. While stirring the water, read times and corresponding temperatures for some three to five minutes before transferring the specimen to the calorimeter. This gives the portion AB of the curve, Fig. 24.

Second period. At a given instant transfer the specimen rapidly to the calorimeter. Continue to stir the water and to take temperature readings every quarter or half minute. While the heated specimen is giving up its heat, the water rises rapidly to a maximum temperature t_f. This period is frequently over in fifteen or twenty seconds. The maximum temperature is attained when the rate at which heat is radiated by the water to the air equals the rate at which the water receives heat from the specimen. The temperature may remain stationary at this value for an appreciable length of time. Thereafter, if the water has risen to a temperature above that of the jacket, the loss by radiation exceeds the gain of heat from the specimen. If within a minute after the specimen is dropped into the calorimeter, the temperature of the water does not rise above that of the jacket, the specimen is to be dried and the experiment begun again. This time do not have quite so much water in the calorimeter and do not have the initial temperature quite so low. If the thermometer rises rapidly and then almost at once falls somewhat, the specimen has come too close to the thermometer and the experiment should be begun again.

The observations of this period give the portion BD of the curve, Fig. 24.

Third period. Without interruption continue to stir the water and to take readings of temperature and time for at least five minutes during the cooling of the water in the calorimeter. The observations of this period give the portion DE of the curve, Fig. 24.

After the third period weigh the inner vessel and contents, and so determine the mass of the specimen.

With these readings, plot on the same sheet two curves — one coördinating temperature and time for the water in the calorimeter, and another coördinating temperature and time for the surroundings. A pair of such curves is shown in Fig. 24. The

distance corresponding to hf, Fig. 24, represents the temperature change $(t_f' - t_i')$ that would have occurred if the calorimeter had neither gained heat from nor lost heat to the surroundings. The water equivalent of the calorimeter and accessories is to be computed by (109).

The data are now at hand which when substituted in (129) give a value for the specific heat of the specimen.

The first time this experiment is performed it may be somewhat shortened by having the initial temperature of the water in the calorimeter at the temperature of the surrounding air. The observations of the temperatures of the space between the inner and outer vessel may also be omitted. The curves obtained correspond to the portion CDE, Fig. 24.

Exp. 76. Determination of the Specific Heat of a Liquid by the Method of Cooling

Theory of the Experiment. — Read Arts. 28, 29, and 30. Suppose that a mass m_l of a liquid of a specific heat s_l is contained in a vessel which has a water equivalent e and a radiation constant r. If the temperature of the liquid is somewhat above that of the surroundings, and if the temperatures of both liquid and surroundings are observed for some little time, and then the temperatures are plotted against times, two curves like CD in Fig. 22 will be obtained. While the liquid and vessel fall in temperature through a range of $\Delta t_l°$, the heat which they lose is, from (106),

$$H_l = (m_l c_l + e c_w)\, \Delta t_l. \tag{130}$$

Since this heat is lost by radiation, (111) shows that it is also given by

$$H_l = rkA_l, \tag{131}$$

where r, k, and A_l have the same meanings as the r, k, and A_{CF} in (111). From (130) and (131) we have at once

$$(m_l c_l + e c_w)\, \Delta t_l = rkA_l. \tag{132}$$

In the same way, if the liquid in question is replaced by warm water, and if subscripts w mean that the symbols to which they are appended refer to this water,

$$(m_w c_w + e c_w)\, \Delta t_w = rkA_w. \tag{133}$$

If the scales chosen in plotting are the same for both pairs of curves, the k in (132) and (133) is the same; and if the nature of the surface of the containing vessel remains the same, the r is the same. On dividing each member of (132) by the corresponding member of (133) we have

$$\frac{c_l}{c_w}\,[= s_l] = \frac{A_l \Delta t_w (m_w + e)}{A_w \Delta t_l m_l} - \frac{e}{m_l}. \qquad (134)$$

Manipulation. — The apparatus used in this experiment consists of a closed metal radiating vessel suspended in an inclosure surrounded by an ice jacket. The radiating vessel is provided with a stirrer for agitating its contents and a thermometer for reading temperatures.

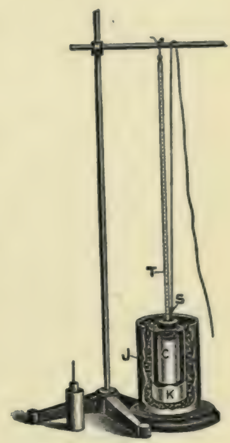

Fig. 39.

Weigh the radiating vessel and stirrer and, by multiplying their mass by the specific heat * of the material of which they are composed, determine their water equivalent. Fill the radiating vessel just to the bottom of the neck with the liquid whose specific heat is to be determined and set it in water in a saucepan over a burner until its temperature is about 40° C. Then wipe the outside of the radiating vessel dry, suspend it in the inclosure inside the ice jacket, and while continually stirring, observe the temperature every minute for quarter of an hour or longer. Throughout this time keep the jacket full of ice. Then remove the radiating vessel from the jacket and weigh it, thus finding the mass of the liquid.

Clean the vessel, rinse it out with water, and fill to the bottom of the neck with water. Then heat, dry, and suspend in the ice jacket as before, and again observe the temperature every minute for a quarter of an hour. Remove from the jacket and weigh, thus finding the mass of the water.

* If the radiating vessel or stirrer is of unknown composition, the water equivalent can be obtained experimentally by the method of mixtures, Exp. 74.

Plot the cooling curves for both substances on the same sheet. Since the jacket is packed with shaved ice, the temperature of the surroundings in each case is zero. If, then, times are plotted as abscissæ, A_w is the area bounded on the top by the water curve, on the bottom by the temperature axis, and on the sides by any two convenient ordinates, — one near the beginning and one near the end of the time employed, — and A_l is the same area except that its upper boundary is the other curve. These areas may be obtained with a planimeter, determined by counting the millimeter squares, or, perhaps most easily, found by the method of average ordinates. Δt_w is the difference between the temperatures at the points where the water curve crosses the ordinates which bound the area A_w, and Δt_l is the corresponding difference for the other curve.

All of the data are now at hand for calculating the specific heat of the specimen by means of (134). Two or three cooling curves for each substance should be taken, and two or three values for the specific heat thus obtained.

Exp. 77. Determination of the Heat Equivalent of Fusion of Ice

Theory of the Experiment. — Read Arts. 28–30. The Heat Equivalent of Fusion of a substance is the number of heat units required to melt unit mass of it without changing its temperature.

Suppose that when m_i grams of ice at 0° C. are dropped into m_w grams of water at t_w°, the ice melts and the temperature of the mixture of the two becomes t_2°. During this operation, the ice has absorbed the heat required to melt it and also after melting to raise its temperature from 0° to t_2°, while the calorimeter and its contents have lost heat. If there were no gain of heat from the surroundings nor loss to them, the heat gained by the ice in melting and then rising to the temperature t_2 would equal the heat lost by the calorimeter and contained water. That is, if e denotes the water equivalent of the calorimeter, and L_f the number of heat

units required to melt unit mass of ice, we should have, from (106) and (107),

$$m_iL_f + m_ic_w\,(t_2 - 0) = (m_wc_w + ec_w)\,(t_w - t_2).$$

That is, the heat equivalent of fusion would be

$$L_f = \frac{(m_w + e)\,(t_w - t_2)\,c_w}{m_i} - t_2c_w. \tag{135}$$

In most cases, however, the error due to radiation is too great to be neglected. This error may either be computed by Regnault's method or determined graphically by the modification of Rowland's method given in Art. 30. If the latter method be selected, it is necessary to determine the temperature change of the mixture that would have occurred if there had been no radiation or absorption. Denoting this corrected value by $(t_w' - t_2')$, corresponding to the distance hf, Fig. 24, we obtain the corrected equation

$$L_f = \frac{(m_w + e)\,(t_w' - t_2')\,c_w}{m_i} - t_2'c_w. \tag{136}$$

The simple theory given in this experiment applies only to a solid whose temperature is at its melting point at the moment it is introduced into the calorimeter. In the general case not only will the temperature of the specimen be below its melting point at the moment of its introduction into the hot water of the calorimeter, but in addition its specific heat will be different in the solid and the liquid states. Even though neither of these specific heats is known, by means of three experiments, similar to the above, in which the masses of the specimen and the water, as well as the original temperature of the water, are different, the heat equivalent of fusion of a substance can be found. We have thus three simultaneous equations containing but three unknown quantities, *viz.*, the required heat equivalent of fusion and the specific heats of the specimen in the solid and in the liquid states. By eliminating the specific heats, the heat equivalent of fusion can be determined.

Manipulation. — Weigh the inner vessel of the calorimeter and the stirrer. The product of their mass and the specific heat of the material of which the vessel and stirrer is composed gives the water equivalent e. Fill the vessel about two-thirds full of water at

about 10° above the room temperature, weigh, and then assemble the calorimeter.

Cut a piece of ice having a mass from one-fifth to one-sixth of that of the water contained in the calorimeter. Keeping the water in the calorimeter well stirred, read the temperature of the water about every half minute and of the surroundings every minute or two. Record the hour, minute, and second at which each reading is made. At a given instant, after reading temperatures for four or five minutes, drop the carefully dried ice into the

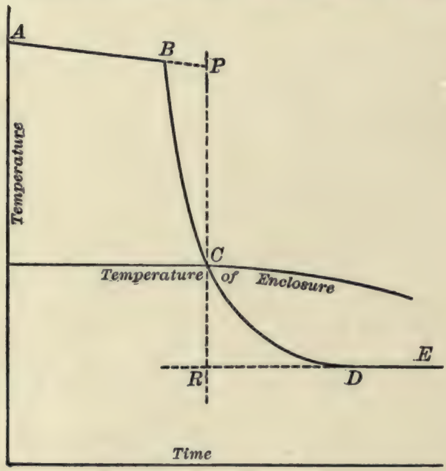

FIG. 40.

calorimeter and continue reading temperatures and stirring for seven or eight minutes longer. The ice must be kept submerged, and under no circumstances must the temperature of the inner vessel fall so low that dew forms on it. Now weigh the inner vessel with its contents. The data for determining m_w and m_i are now at hand.

The corrected temperature of the mixture can be determined graphically, as follows: on a single pair of coördinate axes plot two curves — one coördinating temperature and time for the water in the calorimeter, and the other for the air between the two vessels. Such a pair of curves is shown in Fig. 40. Through the

point of intersection of the two curves draw a line parallel to the temperature axis. Produce the cooling curve AB until it intersects this line PC at some point P. Prolong ED backward until it intersects the line PC at some point R. The point P represents the temperature to which the water and the calorimeter would have fallen at the time represented by the abscissa of C, if no ice had been added. The point R represents the temperature to which the mixture would have fallen if no heat had been absorbed from the surroundings. Therefore, the ordinate of P is the t_w' of (136), and the ordinate of R is t_2'.

It may be necessary to make one or two preliminary experiments to determine just how warm to have the water and just how much ice to use. After the experiment has been performed successfully it should be repeated once or twice, two or three values for the heat equivalent of fusion being thus obtained.

Exp. 78. Determination of the Heat Equivalent of Vaporization of Water

THEORY OF THE EXPERIMENT. — Read Arts. 28–30. If heat be applied to a liquid, the liquid rises in temperature until its maximum vapor pressure becomes a trifle greater than the external pressure on its surface. The vapor pressure is then great enough to make the bubbles in the liquid expand in spite of the pressure of the liquid outside of them. As the bubbles grow they rise to the surface and burst and the liquid is said to boil. Further addition of heat does not raise the temperature, but simply makes the evaporation into these bubbles go on faster, *i.e.*, produces more rapid boiling. The number of heat units required to vaporize unit mass of a liquid is called the *heat equivalent of vaporization* of the liquid. The object of this experiment is to determine the heat equivalent of vaporization of water.

Let m_s grams of steam be condensed in m_w grams of water contained in a calorimeter of water equivalent e. Let t_s denote the temperature of the steam; t_w, the temperature of the calorimeter and contents at the moment the steam began to enter; t_2, the temperature of the two after they are mixed; and L_v, the heat equiva-

lent of vaporization of water. Then the heat given up by the steam in condensing and then cooling to the temperature t_2 equals the heat taken up by the calorimeter and contents plus the heat lost by radiation. That is, from (106) and (107),

$$m_s L_v + m_s c_w (t_s - t_2) = (m_w c_w + e c_w)(t_2 - t_w) + R,$$

where R is the radiation correction. Whence,

$$L_v = \frac{(m_w + e)\, c_w (t_2 - t_w) + R}{m_s} - (t_s - t_2)\, c_w. \qquad (137)$$

MANIPULATION. — The apparatus used in this experiment comprises a boiler in which the liquid is vaporized and a calorimeter containing a copper worm in which the vapor is condensed. The liquid in the boiler A (Fig. 41) is heated by means of an electric current passing through a coil of wire. The arm holding the boiler is attached to a vertical rod supported by the tubular column B. Below the clamp D there is a horizontal slit extending through an arc of about 90°, and from one end of this horizontal slit there is a vertical slit extending about halfway down the tubular column. A pin in the vertical rod supporting the boiler extends through this slit. By means of this arrangement, the boiler can be rotated quickly into a definite plane and dropped in a vertical line so as to cause the outlet O of the boiler to register with the end W of the copper worm contained in the calorimeter C.

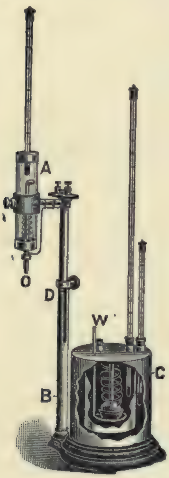

FIG. 41.

Weigh separately the condensing worm and the inner vessel of the calorimeter with the stirrer, and determine their total water equivalent e. Pour water at about 10° below room temperature into the inner vessel of the calorimeter until all the convolutions of the condensing worm are covered. Determine the mass m_w of this water. Assemble the apparatus and adjust the position of the calorimeter until the outlet of the boiler will register accurately with the opening in the rubber stopper on the end of the condensing

worm. Raise the boiler, thus disconnecting it from the calorimeter, rotate it to one side, and pour into it enough distilled water to cover all the turns of the wire. Connect a 110-volt circuit to the terminals of the wire spiral and adjust the rheostat until the water boils rapidly but does not spatter over into the outlet tube.

Now commence stirring the water in the calorimeter, every half minute recording its temperature, and every minute or two recording the temperature of the air in the jacket. Record the hour, minute, and second at which each reading is made.

After reading for two or three minutes rotate the boiler into position, drop it into place, and, without interrupting the stirring and reading of temperatures, allow steam to flow into the condensing worm until the temperature of the water in the calorimeter rises to 40° or 45°. Disconnect the boiler from the calorimeter, rotate it to one side, throw off the current, and continue stirring and taking temperature readings at one minute intervals for about ten minutes. Remove the condensing worm from the calorimeter, carefully dry the outside, and weigh. The difference between this mass and the mass of the worm, already determined, is the mass m_s of the condensed steam. Read the barometer, correct it as indicated in Art. 27, and by Table 3 find the temperature of the steam.

Compute the value of the radiation correction R by Regnault's method in the manner given in Art. 30a. In determining this correction notice that the e' in (114) is the water equivalent of everything that cooled along the curve CD (Fig. 23). In the present case e' is the water equivalent of the inner vessel of calorimeter, contained water, stirrer, thermometer, worm, and condensed steam.

Exp. 79. Determination of the Heat Value of a Solid with the Combustion Bomb Calorimeter

THEORY OF THE EXPERIMENT. — Read Arts. 28–30. The object of this experiment is to determine the amount of heat developed by the complete combustion of a unit mass of coal. The heat value of a solid or liquid is expressed either in British thermal units per pound or in calories per gram.

The method to be employed in this experiment is to burn a known mass of the given substance in a strong steel bomb filled with oxygen under high pressure. During the combustion the bomb remains immersed in a water calorimeter and the heat developed is obtained by the ordinary method of mixtures. Thus suppose that by the combustion of m grams of the substance, the bomb together with the calorimeter, its accessories, and the contained water rise in temperature from $t_1°$ to $t_2°$ C. If the mass of water in the calorimeter is m_w grams, the total water equivalent of calorimeter, bomb, thermometer, and stirrer is e grams, and the radiation correction is R calories, then the heat value of the substance is

$$H = \frac{(m_w c_w + e c_w)(t_2 - t_1) + R}{m} \text{ calories per gram.} \qquad (138)$$

The superiority of this method is that since in it complete combustion is attained and all the products of the combustion remain in the apparatus, the quantity of heat developed is readily computed.

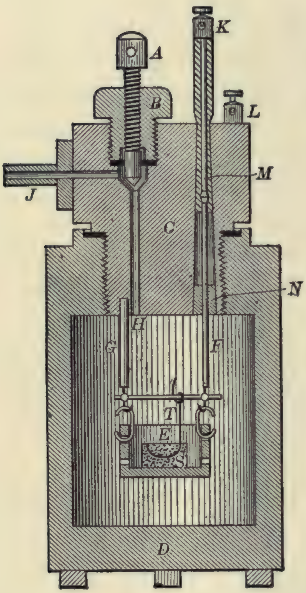

FIG. 42.

MANIPULATION. — The apparatus used in this experiment consists of a water calorimeter, a combustion bomb, a press for molding the specimen into a small coherent pellet. If compressed oxygen is not available it must be generated by some such device as that shown at R, Fig. 45.

Hempel's combustion bomb consists of a soft steel or cast-iron capsule D (Fig. 42), closed by a massive plug C. The inside surface of the bomb is coated with enamel. The plug is pierced by two passages — one JH for filling the bomb with oxygen, and the other for the introduction of an insulated conductor KF. The gas passage is controlled by the compression valve A. The rod KF

is insulated from the metal plug by the rubber packing *M* and asbestos packing *N*. *G* is a metal rod screwed into the plug. A little basket *E*, made of incombustible material, is suspended by means of heavy platinum wires from the ends of the rods *G* and *F*. The ends of the rods *G* and *F* are connected by a thin platinum wire.

In preparing a specimen of pulverulent coal for a determination, the coal is first ground in a mortar and then molded into a compact coherent pellet by means of a screw press (Fig. 43). The mold of the press consists of a block of steel *X* (Fig. 44) bored out to the

Fig. 43.

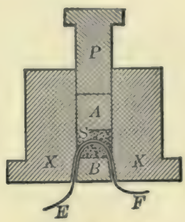

Fig. 44.

required size. The upper portion of this hole is cylindrical and is fitted with a cylindrical plug *A*. The lower portion of the hole is reamed out to a conical form and is fitted with a conical plug *B*. On the conical surface of the plug are two narrow channels which extend from one face to another. Loop a piece of thread *FSE* over the conical plug and lay its end in these slots. Pack about 1.25 g. of pulverized coal about this loop of thread, put the cylinder *A* in place and the plunger *P* on top of it, and turn the screw down until the specimen has been compressed into a compact pellet. Raise the screw, slip the mold into the upper horizontal guides, and again depress the screw until the pellet is forced out through the bottom of the mold. With a sharp knife pare down the pellet until it weighs about one gram. Cut off one end of the thread close to the specimen. Remove any loose particles of coal by means of a small brush, place the pellet on a watch glass, and weigh.

Do not touch the pellet with the fingers, but handle by means of the thread.

In the case of anthracite or other coal of which a coherent specimen of the proper size can be obtained, the string can be tied around the specimen instead of imbedding it in a compressed pellet.

In case that a tank of compressed oxygen is not available, unscrew the plug *C* (Fig. 42) of the bomb, mount it in a retort stand, connect the terminals of an electric circuit to the binding posts *K* and *L*, and carefully decrease the resistance in the circuit until the

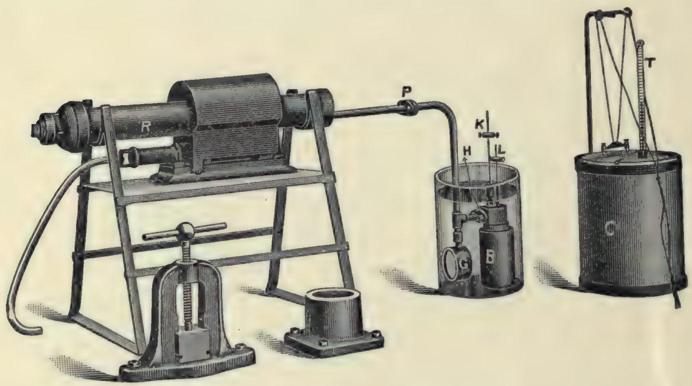

Fig. 45.

current will just bring the platinum wire connecting *G* and *F* to a red glow. Without disturbing the resistance in the circuit, open the switch and disconnect the terminals from the binding posts *K* and *L*. Place the specimen of coal in the basket *E* and tie the free end of the thread to the wire connecting *G* and *F*. Without disturbing the specimen, remove the plug from its support and screw it tightly into the bomb. The bomb is now ready to be filled with oxygen. Into the gas generating retort *R* (Fig. 45) put a mixture of about two hundred grams of potassium chlorate and some fifty grams of manganese dioxid. Put a tightly wound roll of copper or brass wire gauze into the tube leading from the retort, and connect the retort and a pressure gage *G* to the combustion bomb in the manner shown in the figure. Before beginning to heat, shake

the retort so as to spread the mixed potassium chlorate and manganese dioxid along its whole length. The pressure gage and combustion bomb are immersed in a vessel of water for the purpose of detecting any leak in the bomb and also for the purpose of cooling the oxygen coming from the hot retort.

Open the gas valve in the combustion bomb and apply a gas flame near the farther end of the gas generating retort until the gage indicates a pressure of about 1 Kg. per sq. cm. (14 lb. per sq. in.). If the flame be now removed, the heat already given to the retort will generate enough oxygen to raise the pressure to about 5 Kg. per sq. cm. (70 lb. per sq. in.). Now loosen the flange coupling P so as to allow the mixture of oxygen and air contained in the apparatus to escape. By tightening the coupling P and repeating this operation the entire apparatus can be freed of air. Now tighten the couplings and slowly heat the retort until the gas pressure rises to about 12 Kg. per sq. cm. (170 lb. per sq. in.). Close the gas valve on the combustion bomb and immediately afterwards disconnect the bomb at the coupling H from the remainder of the apparatus. Cool the bomb to about the temperature of the room and carefully dry it with a towel.

If compressed oxygen is available connect the bomb to the oxygen tank through a pressure reducing valve and pressure gage. The pressure inside the bomb should be about 170 lb. per sq. in.

Place the bomb in a water calorimeter C (Fig. 45) containing m_w grams of water at about the room temperature. Connect the terminals of the previously arranged electric circuit to the binding posts K and L, and see to it that K and L are not short-circuited by the cover of the calorimeter. Before closing the switch in the electric circuit take temperature readings of the continuously stirred water at half-minute intervals for at least five minutes. At a given instant close the switch so that the electric current will ignite the specimen. The switch should be closed for a moment only or the heating effect of the current will need to be taken into account. Continue stirring the water and taking half-minute temperature readings for at least ten minutes after ignition. Take the bomb out of the water, open the valve, unscrew the head, wash out the inside, and oil the screw threads.

From a curve coördinating temperature and time find by Rowland's method described in Art. 30*b* the temperature change of the calorimeter that would have occurred if there had been no loss of heat by radiation. Let $(t_2'' - t_1'')$ represent this corrected temperature change, represented in Fig. 24 by the distance *hf*. Then instead of (138) we can write

$$H = \frac{(m_w c_w + e c_w)\,(t_2'' - t_1'')}{m} \text{ calories per gram.} \qquad (139)$$

In this equation the water equivalent e is still unknown. This can be determined in any of three ways: (*a*) By taking the sum of the products of the masses and the assumed specific heats of the various parts of the apparatus. In an apparatus like this, composed of so many different materials of uncertain composition, this method is unreliable. (*b*) Experimentally, by the method of mixtures. The large amount of water required in this experiment and the difficulty of obtaining temperatures accurately make this method unsatisfactory for inexperienced observers. (*c*) By means of a supplementary experiment in which a definite amount of heat is developed in the apparatus by the combustion of a known mass of a substance having a known heat value. There are a number of substances the heat values of which are accurately known and which can easily be obtained pure. The last method is the one that will be employed in this experiment.

Suppose that when using the same apparatus as before, the burning of m' grams of a substance of heat value H' raises the temperature of the apparatus and of m_w' grams of water from t_1'° to t_2'° C. Let $(t_2''' - t_1''')$ be the temperature change of the calorimeter that would have occurred if there had been no loss of heat by radiation. Then

$$H' = \frac{(m_w' c_w + e c_w)\,(t_2''' - t_1''')}{m'} \text{ calories per gram.} \qquad (140)$$

Whence, on solving for e,

$$e = \frac{m'H'}{c_w\,(t_2''' - t_1''')} - m_w'. \qquad (141)$$

Naphthalin is a suitable substance to use in this supplementary experiment. Make a pellet of somewhat smaller mass

than that of the coal already used and proceed exactly as in the experiment with the coal. Use (141) to find e, and then (139) to find H.

Before putting away the apparatus dig the remaining solid substance out of the gas retort, rinse out the combustion bomb with water, and carefully oil the threads of the bomb and all parts of the press. Be certain that no water or oil is left inside of either the retort or the bomb. If oil or any other organic substance is heated in the retort with the oxygen producing mixture an explosion is liable to occur.

Exp. 80. Determination of the Heat Value of a Gas with Junker's Calorimeter

THEORY OF THE EXPERIMENT. — Read Art. 28. The object of this experiment is to determine the number of heat units developed by the combustion of unit volume of a given sample of gas. In Junker's method the heat developed by a steady flame is determined by measuring the heat absorbed by a steady stream of water inclosing the flame.

The apparatus consists of an accurate gas meter M (Fig. 46), a gas pressure regulator R, and a calorimeter C, of special design. The calorimeter consists of a combustion chamber A (Fig. 47), inclosed by a water jacket B, traversed by a large number of tubes for the passage of the products of combustion. The water jacket is surrounded by a closed space L filled with air. After traversing the meter and pressure regulator the gas is burned in the burner Q. The products of combustion after passing through the tubes traversing the water jacket escape through the vent Y. The temperature of the gas as it enters the burner and the temperature of the products of combustion as they leave the calorimeter are given by the thermometers T'' and T'''. A stream of water flows from the supply pipe D into a small reservoir kept at constant level by means of the overflow pipe O. From this regulator the water passes down the tube E through the control valve V, thence through the water jacket B, thence through G and the discharge nozzle H into the measuring vessel U. The tem-

peratures of the water as it enters and as it leaves the calorimeter are given by the thermometers T' and T. Water vapor formed by the combustion of the gas condenses on the inside of the combustion chamber and escapes through the outlet J into the measuring vessel W.

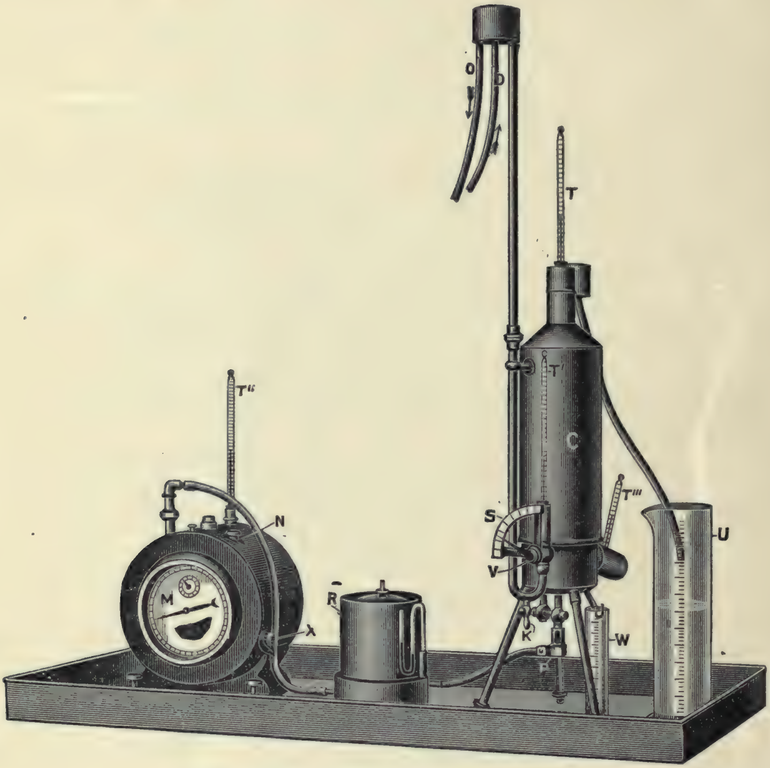

Fig. 46.

Let v represent the volume (reduced to standard conditions) of the gas burned during a certain time. Let the mass of water which passes through the calorimeter during this time be denoted by m_w, and let its temperatures on entering and on leaving be represented by t' and t respectively. Let the mass of steam con-

densed during the combustion be represented by m_s, and let the temperature at which it condenses and the temperature of the condensed steam as it leaves the calorimeter be denoted by t_s and t_c respectively. Represent the mean thermal capacity of water by c_w.

Then the heat value of the gas H is given by the equation

$$H = \frac{m_w c_w (t - t') - m_s L_v - m_s c_w (t_s - t_c)}{v}, \tag{142}$$

where L_v is the heat equivalent of vaporization of water. If m_w and m_s are measured in grams, v in liters, and temperatures in degrees centigrade, then H is given in gram calories per liter or kilogram calories per cubic meter.

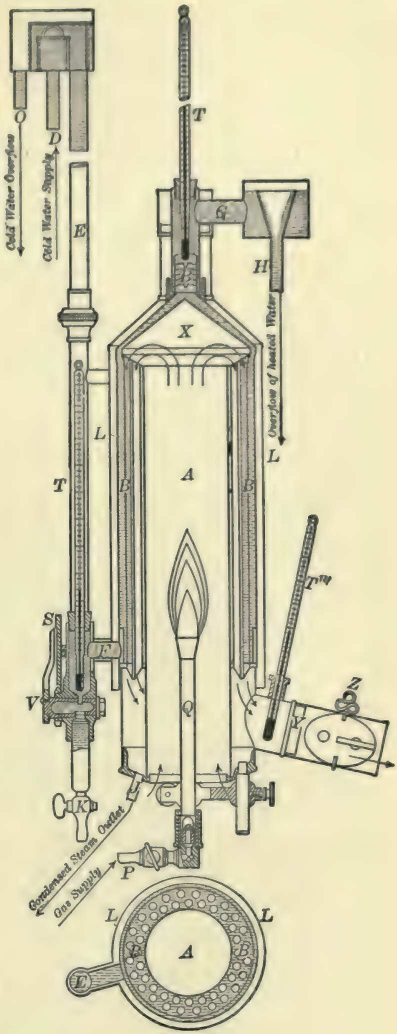

FIG. 47.

MANIPULATION. — After assembling the apparatus, connect D to the water supply so that any leak in the calorimeter will make itself evident. The flow of water into the apparatus must always be sufficiently great to overflow through the pipe O. With gas valve at the burner P closed, connect the gas regulator to the gas supply and notice whether the index of the meter moves. If it does, seek out the leak and remedy it. With

the water still flowing through the apparatus, take the burner out of the calorimeter, light the gas, and replace the burner. If the gas is lighted while the burner is inside the combustion chamber, there is danger of an explosion. Have the top of the burner from 12 to 15 cm. above the lower opening to the combustion chamber. The damper Z should be from one-half to completely open, depending upon the draught required for the flame.

Adjust the flow of gas to about three liters per minute. Adjust the flow of water so that the difference in the temperature of the ingoing and outgoing water shall be about 15° C. Adjust the damper till the temperature of the outgoing products of combustion shall be within 1° C. of the temperature of the ingoing illuminating gas.

After all of the thermometers indicate nearly stationary temperatures, note simultaneously the gas meter reading and the temperatures indicated by the thermometers T and T'. Then immediately place suitable vessels U and W so as to catch the warmed water escaping from H and the condensed steam escaping from J. Note the temperatures of the ingoing and the outgoing water every 15 seconds until two or more liters of water have flowed into the vessel U. Then remove the vessels U and W and at the same time take the gas meter reading. Note the temperature t_c of the condensed steam in W. Determine m_w and m_s by weighing.

From the difference between the two gas meter readings together with the temperature and pressure of the gas passing to the burner, the value of v is found by means of the fundamental law of gases. The temperature is given by the thermometer T''. The pressure is the sum of the barometric reading and the height of mercury corresponding to the difference in the levels of water in the manometer V.

All of the data are now at hand for substitution in (142).

By substituting a properly designed lamp for the gas burner, Junker's calorimeter can be used for finding the heat value of a liquid.

Exp. 81. Determination of the Heat Value of a Gas by Means of Parr's Comparison Calorimeter

THEORY OF THE EXPERIMENT. — The object of this experiment is to determine the heat value of a specimen of illuminating gas by comparison with the heat liberated when an equal volume of another gas of known heat value is burned under the same conditions.

Parr's Comparison Gas Calorimeter includes two calorimeters C_1 and C_2, Fig. 48, of the same volume, thermal capacity, and radiating power. Each consists of a combustion chamber surrounded by a water jacket. The water in each is kept agitated by means of stirrers Q_1 and Q_2 operated by a belt and motor M. The temperatures of the water are indicated to hundredths of a degree by thermometers T_1 and T_2.

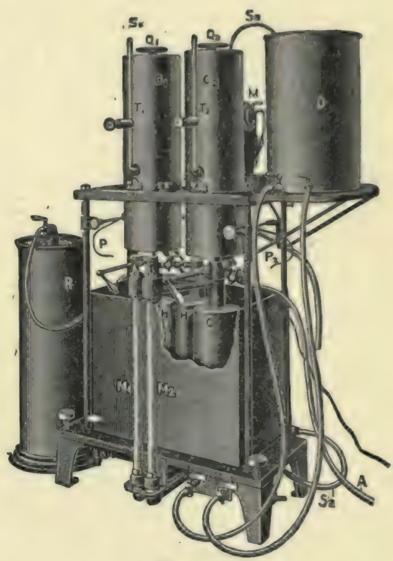

FIG. 48.

The gas that supplies the burners in the calorimeters is contained in three similar cylindrical holders H, H', and G, immersed in the same tank of water. The gas holders H and H' can be so joined that either the contents of one or of both can supply the burner in calorimeter C_1. The holder G is connected to the burner in calorimeter C_2. These holders can be joined to the water tank D and their contents be thereby brought to the same pressure. The height of water in each holder is indicated by the water glasses M_1 and M_2.

The gas under test enters the holder G through the tube A. The gas of known heat value stored in the gas reservoir R enters either one or both of the holders H, H'. In case the gas under test has a

heat value about that of the standard gas, only one holder is used for the standard gas. But if the gas under test has a heat value about double that of the standard, both H and H' are filled with the standard gas. The system of stop cocks permits the gas holders to be filled to the same level, as indicated by the water glasses M_1 and M_2.

Starting with the gases in the holders at the same level, if the burners be lighted when each is connected to a single holder, and the burners be turned off when the gas in the holders has reached a common level, then equal volumes of the two gases at the same pressure and temperature will have been burned. If the heat thereby produced be absorbed by equal masses of water contained in calorimeters of equal thermal capacity, and if the heat lost by radiation and conduction from the two calorimeters be the same, then the ratio of the rise in temperature of the two calorimeters will equal the ratio of the heat values of the two gases, or in symbols,

$$\frac{\Delta t_1}{\Delta t_2} = \frac{H_1}{H_2}. \tag{143}$$

For use as a standard gas, hydrogen is available and convenient. As its heat value is only about one-half that of illuminating gas, two volumes of hydrogen should be used to one of illuminating gas. At 60° F. and 30 inches of mercury the accepted heat value of hydrogen is 325 B.t.u. per cubic foot. As in this method both gases are at the same temperature and pressure, any variation from the above specified temperature and pressure will affect both gases alike, and consequently no error will be introduced.

A sample set of data with the computation is given below:

	Temperature		
	initial	final	rise
Hydrogen (2 vols.)	68.92° F.	73.87° F.	4.95° F.
Illuminating gas (1 vol.)	68.78° F.	73.32° F.	4.54° F.

Remembering that the heat value of hydrogen is 325 B.t.u. per cu. ft., we have from (143),

$$H_2 = H_1 \frac{\Delta t_2}{\Delta t_1} = (2 \times 325) \frac{4.54}{4.95} = 596.16 \text{ B.t.u. per cu. ft.}$$

MANIPULATION. — Fill the calorimeters to the point of overflow and allow the siphons to drain off the excess water. With the filled tank D in place as shown in the illustration, open the valves in the tubes to the gas holders and also the valves at the tops of the holders. Let the holders fill till water escapes through the valves at the tops. Close the valves at the tops, connect the double holder HH' to the hydrogen supply tank, and the remaining holder G to the city gas supply. If now the water tank D be taken off its shelf and held below the level of the table while the gas supply valves are open, the holders will fill with gas. When no water shows in the water glasses, close the valves at the water tank and replace the tank on its shelf. By means of the vent valves at the top of the water glasses, allow gas to escape until water stands in the water glasses at the same level near the middle of the lower series of graduations.

Wait a few minutes to see if there are any leaks. Start the stirring motor and take four or five temperature readings of the two calorimeters at intervals of one minute. By means of the pilot jets P_1 and P_2 light the two burners as nearly simultaneously as possible. Regulate the flow of gas to the burners till water rises at the same rate in the two water glasses, and also the time required for the water to rise to the upper series of graduations is about eight minutes. Turn off the gas when the water has reached corresponding marks on the water glasses. Continue taking temperature readings at one minute intervals as long as the calorimeters rise in temperature. In making the computation, take as the initial temperatures the readings taken just before the gas was lighted; and as the final temperatures, take the maximum values. These maximum temperatures will occur two or three minutes after the gas was turned off.

Make not less than three determinations.

As the final temperature of one calorimeter is higher than that of the other, one calorimeter will lose more heat by radiation, conduction, and convection than the other. The error thereby introduced, however, is well within the tolerance of industrial requirements. This error can be reduced by having the initial temperature of the water in the calorimeters as much below the temperature of the room as the final temperature is above.

Care should be taken that water does not get into the burners, or unburned gas into the combustion chambers. The valves to the burners should be opened only when it is desired to ignite the gas. To ignite the gas, first turn the pilot flames into the combustion chambers, then quickly turn the burner valves 45° or more, and rotate the pilot flames out of the combustion chamber. The burners and flames can be viewed with the aid of a hand mirror.

Pure compressed hydrogen is supplied in tanks ready for use. To fill the hydrogen supply reservoir R, the dome is first filled with water and then the water is displaced by hydrogen from the pressure tank.

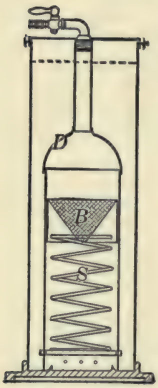

FIG. 49.

When compressed hydrogen is not at hand, hydrogen can be generated in the reservoir R by bringing water into contact with an alloy of sodium and lead. This alloy is marketed under the trade name "hydrone." To charge the reservoir for use as a hydrogen generator, remove the dome D, Fig. 49, take out the spring S and basket B, and wipe dry the basket, spring and inside of the dome. Put into the basket one or two pounds of "hydrone" and assemble the apparatus. Displace the air within the entire reservoir by illuminating gas. Now fill the outer tank with water. On opening the valve at the top of the dome, water will rise in the dome, and, on coming into contact with the "hydrone," hydrogen will be generated. Open the valve at the top of the dome till the escaping hydrogen has cleared the dome of illuminating gas. The generator is now ready for use.

Make three determinations of the heat value of the illuminating gas and express the mean result in British Thermal Units per cubic foot at 60° F. and 30 in. of mercury pressure.

Exp. 82. Determination of the Economy Effected by Steam Pipe Covering

THEORY OF THE EXPERIMENT. — From an uncovered steam pipe much heat is lost by convection and radiation. This loss is much diminished by enclosing the pipe in various pipe coverings

or laggings made of poorly conducting materials. The object of this experiment is to determine the number of heat units saved in unit time from unit area of a pipe when at a given temperature above that of the surroundings by covering the pipe with a given brand of lagging.

The method involves the use of two pieces of piping, exactly similar except that one is entirely enclosed in covering of the particular brand under test, while the other is left uncovered for the length of a standard section of the covering under test. The pipes are filled with oil which can be heated by current-carrying coils. The current in each coil is adjusted till the temperature of the oil in the two pipes has the same predetermined constant value, say 100° or 200° C. The difference between the amounts of power now being supplied to the two heating coils equals the saving of power effected by one standard length of the given brand of pipe covering.

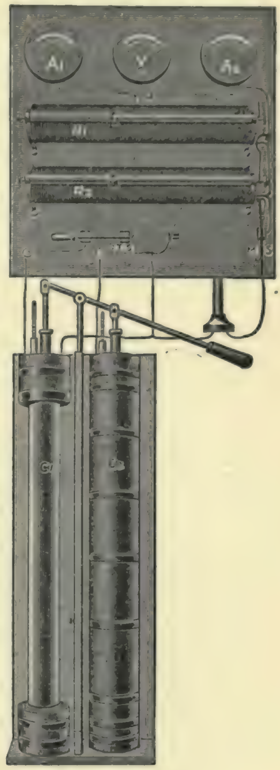

FIG. 50.

The apparatus consists of two pieces of 3-in. pipe of equal length covered alike with a thick layer of magnesia and asbestos except for an equatorial space of a length equal to that of a standard section of pipe covering. A vertical board between the pipes prevents heat from one pipe affecting the other. The empty equatorial space about one pipe is filled by a section of the covering to be tested, Fig. 50. In each pipe is a heating coil wrapped about an iron tube fastened in the axis of the pipe. This tube also serves as the cylinder for a pump by means of which the oil in the pipe is thoroughly stirred.

The heating coil C_1 in one pipe is joined in series with an adjust-

able rheostat R_1 and ammeter A_1, Fig. 51, while the heating coil C_2 in the other pipe is joined in series with a rheostat R_2 and ammeter A_2. By means of a double-throw-switch DTS, a voltmeter V can be connected to the terminals of either heating coil C_1 or C_2.

MANIPULATION. — Measure the length and diameter of the uncovered pipe.

Close the main switch MS and adjust the rheostats R_1 and R_2 till the current in each heating coil C_1 and C_2 is about 10 amperes. While stirring the oil by means of the pumps, observe the temperatures of the oil in the two pipes. When the temperature is about 100° C. in each, regulate the current in each heating coil so as to keep the temperature of the two pipes at a constant value. After the two pipes have been at about the same temperature for ten minutes, note the temperature of each oil bath and the current in each coil. By throwing the handle of the double-throw-switch DTS first to one side and then to the other find the potential difference at the terminals of each heating coil.

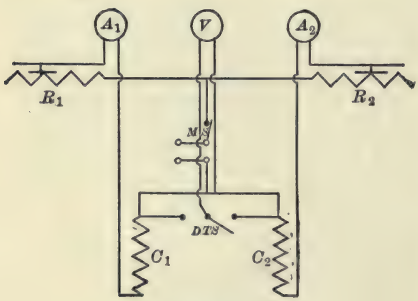

FIG. 51.

Take similar readings at one minute intervals for ten minutes.

Using the averages of the readings, compute the power expended in each pipe, expressed in watts, and also in calories per second and in British thermal units per hour. Also compute the power saved by the use of the given brand of pipe covering, expressed in horsepower per square foot of pipe surface.

Exp. 83. Determination of the Mechanical Equivalent of Heat by Rowland's Method

THEORY OF THE EXPERIMENT. — Read Arts. 24, 28–31. One method of determining the mechanical equivalent of heat is to measure the amount of heat developed when a given amount of

mechanical energy is used to stir water vigorously. In the apparatus used by Joule and improved by Rowland this stirring is done in the inner vessel C (Fig. 52) of a calorimeter. From the inner walls of this vessel project vanes VV between which the paddles PP have just room to turn. These paddles are fastened to a piece

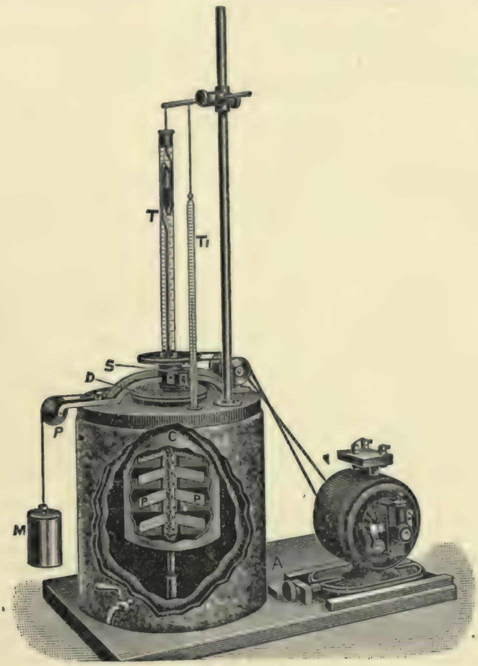

FIG. 52.

of brass tubing that carries at its upper end a disk which is driven by the belt from the small motor seen at the right. The vessel C is supported below on a point with very little friction and on top carries a disk D. Around this disk is lapped a cord which passes over a pulley P and carries at its end a mass M, If there were nothing to prevent it, the weight of M would cause C to turn until a projection on D came against one of two stops between which it plays. But the motion of the paddles PP throws water against the vanes VV so rapidly that when the adjustments have been

properly made C remains nearly at rest, the projection on D playing between the two stops.

Let M denote the mass of M and d the diameter of D. Then $Mg \times \frac{1}{2} d$ is the torque that M exerts in keeping C from turning with the paddles. When the paddles have turned n times they have turned through an angle of $2\pi n$ radians. From the proposition in elementary dynamics which states that the work done by a rotating body is measured by the product of its rotation in radians and the torque which opposes that rotation, we have then

$$W = \pi n M g d, \tag{144}$$

where W denotes the mechanical energy used in stirring the water.

Let m denote the mass of water in the calorimeter; e, the water equivalent of the vessel C, the paddles, and the immersed part of the thermometer; t_1, the initial temperature of the water in the calorimeter; t_2, its temperature after the paddles have made n turns; c_w, the mean thermal capacity of water, and R the net amount of heat lost from C by radiation. Then from (106),

$$H = c_w (m + e) (t_2 - t_1) + R, \tag{145}$$

where H denotes the amount of heat developed by the churning of the water.

From (115), (144), and (145) we have

$$J = \frac{\pi n M g d}{c_w (m + e) (t_2 - t_1) + R}. \tag{146}$$

Manipulation. — In order to determine the radiation correction R it is necessary to know how the readings of the two thermometers compare. Adjust the Beckmann thermometer, and suspend both thermometers in a bath of water at a temperature nearly equal to that of the room. Stir the water occasionally, and after a time record the reading of each thermometer. Meantime take the diameter of D with a caliper and meter stick, see that the pulley P runs easily, and be sure that the vessel C and the paddles are dry and weigh them together. Then fill C to within a few millimeters of the top with water at a temperature some 3° or 4° below the temperature of the room, and weigh again. Assemble the

apparatus, set the motor running, and by means of the screw A move the motor until the tension of the belt is such as to keep the projection on D playing about halfway between its stops.

At some chosen instant read the thermometer T and immediately thereafter the revolution counter S. For some ten or fifteen minutes after that instant read T and T_1 every minute — always reading one of them half a minute after the other. At the end of this time open the switch that supplies power to the motor, note the reading of the revolution counter, and continue reading the thermometers for five or ten minutes. During this time the water in the calorimeter ought to be kept stirred. This can be done by turning the paddles steadily and very slowly by hand. The paddles must not be turned faster than about one revolution in two minutes.

On the same sheet plot two curves coördinating temperature and time — one for the thermometer T and the other for T_1 — and by Regnault's method determine R. In finding n note that the revolution counter reads 1 for every four turns of the paddles. Determine M by weighing and e by (109).

Without throwing out the water or repeating the weighings make three determinations and find the mean.

Exp. 84. Determination of the Mechanical Equivalent of Heat with Barnes' Constant Flow Current Calorimeter

Theory of the Experiment. — In text-books on General Physics it is shown that when a steady electric current of I amperes flows from one to the other of two points between which there is a potential difference of V volts, in t seconds there is transformed between those two points from electric energy into heat the amount of energy

$$W = IVt \text{ joules} = IVt \cdot 10^7 \text{ ergs}. \tag{147}$$

If the current, the potential difference, the time, and the heat produced can be accurately measured, this equation suggests a method of determining the mechanical equivalent of heat.

In this experiment the electric current flows through a wire

coiled inside of a glass tube B_1B_2 (Fig. 53). Through this same tube is a steady flow of water. The heat developed by the electric current warms the water during its passage through the tube so that the thermometer T_1 indicates a higher temperature than T_2. If m grams of water escape in t seconds, and during the passage through the tube are raised from temperature t_2 to temperature t_1,

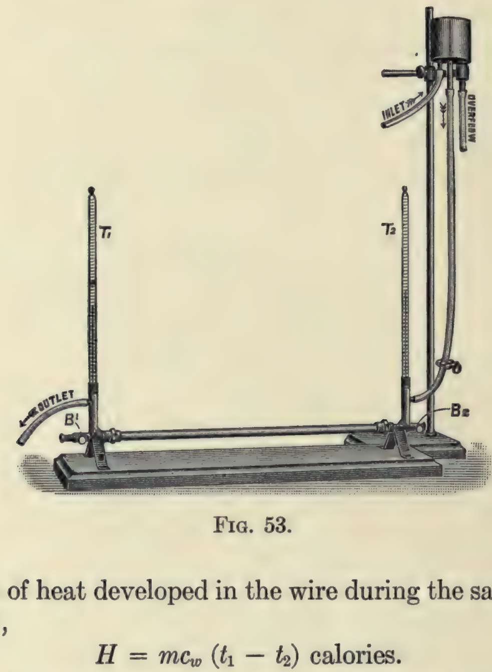

Fig. 53.

the amount of heat developed in the wire during the same t seconds is, by (106),

$$H = mc_w (t_1 - t_2) \text{ calories.} \tag{148}$$

On substituting in (115) the values of W and H from (147) and (148) we obtain

$$J = \frac{IVt \cdot 10^7}{mc_w (t_1 - t_2)} \text{ ergs per calorie.} \tag{149}$$

Since the temperatures t_1 and t_2 are practically steady during the time observations are being taken there are no corrections for the thermal capacity of wire, glass tube, thermometers, nor anything else. With a good flow of water and the mean of the tem-

peratures of the inflowing and outflowing water within 5° C. of the room temperature the heat losses by conduction and radiation are negligible.

MANIPULATION. — The rate at which water flows is kept constant by a small reservoir inside a larger jacket shown at the top of Fig. 53. The water supply is so arranged that water is always overflowing gently from the small reservoir, and the head of water is therefore kept constant.

After the apparatus is set up as shown in Fig. 53 and the water is started, the electric connections are to be made as indicated in Fig. 54. B_1 and B_2 are the binding posts shown in Fig. 53, and W

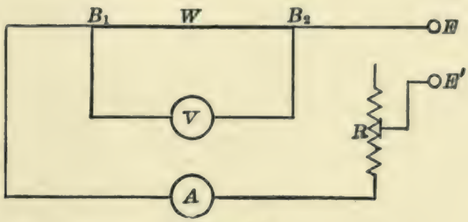

FIG. 54.

is the wire inside the glass tube. V is a voltmeter, A an ammeter, R a rheostat, and EE' the terminals of an electric circuit. In connecting the ammeter and voltmeter care must be taken that the positive wire is connected to the side marked +.

After making the electric connections and adjusting the flow of water and the electric current to suitable values, open the switch in the electric circuit. After a few minutes, when the readings of the thermometers have become steady, record their readings every minute for four or five minutes. Make all thermometer readings to hundredths of a degree. Close the switch, and when the thermometers have again become steady, put under the outlet a weighed vessel and at the same instant start a stop-watch. After fifteen seconds read the voltmeter, after fifteen more the ammeter, after fifteen more one thermometer, and after fifteen more the other thermometer. Continue taking readings in the same order every fifteen seconds for five or ten minutes. At the

end of this time remove the vessel from under the outlet and at the same instant stop the watch. Find the mass of the water that flowed through. To get $(t_1 - t_2)$, subtract the difference between the averages of the temperatures indicated by the two thermometers before the electric current was turned on from the difference between their average readings while the current was flowing.

Take five sets of observations for different rates of flow of water and different values of electric current.

CHAPTER IV

ELECTRICITY AND MAGNETISM

32. Definitions of Units. — (*a*) The *Unit Magnetic Pole* is that pole which if placed at a distance of one centimeter, in air, from another equal and similar pole, will be repelled with a force of one dyne.

(*b*) The unit magnetic field is that field which will act on a unit pole situated in that field with a force of one dyne. An intensity of magnetic field of one dyne per unit pole is called a *Gauss.*

(*c*) The unit of magnetic flux is the flux through one square centimeter perpendicular to a uniform magnetic field of one gauss and is called a *Maxwell.*

(*d*) The *Absolute Electromagnetic Unit of Current* is that current which, if flowing along a conductor one centimeter long, in and perpendicular to a uniform magnetic field of one gauss, will be pushed aside with a force of one dyne.

The practical unit of current is 10^{-1} as large as the absolute unit and is called the *Ampere.*

The *International Ampere* is the unvarying current which, when passed through an aqueous solution of silver nitrate in accordance with the specifications in the following paragraph, deposits silver at the rate of 0.00111800 gram per second.

The cathode shall take the form of a platinum bowl not less than 10 cm. in diameter and 5 cm. deep. The anode shall be a plate of pure silver of 30 sq. cm. in area and two or three millimeters thick. This is to be supported horizontally near the upper edge of the bowl by platinum wires. This anode must be wrapped with filter paper to prevent silver from falling into the bowl. The electrolyte shall consist of 15 parts of pure nitrate of silver to 85 parts of distilled water by weight. The resistance of the metallic circuit shall not be less than 10 ohms.

The method of taking measurements is as follows: Wash the bowl first with nitric acid and then with distilled water. Dry by heating. Afterward cool in a desiccator. When at room temperature weigh and fill nearly full with the electrolyte. Connect to the circuit by placing the bowl on a clean copper ring to which a binding post is soldered. Immerse the anode in the solution, close the circuit, noting the hour, minute, and second. In not less than one-half hour break the circuit, again noting the hour, minute, and second. Empty the bowl, rinse with distilled water, and allow the bowl to soak in distilled water for at least six hours. Rinse with absolute alcohol, dry in a hot air bath, cool in a desiccator, and weigh. The gain in mass is the amount of silver deposited.

Then, the current

$$I = \frac{mt}{0.001118} \text{ amperes,} \tag{150}$$

where m is expressed in grams and t in seconds.

(*e*) The *Absolute Electromagnetic Unit of Quantity* is that quantity of electricity passing a fixed point in a conductor per second when one absolute unit of current is flowing.

The practical unit of quantity is 10^{-1} as large as the absolute unit and is called the *Coulomb*.

(*f*) The *Absolute Electromagnetic Unit of Resistance* is that resistance which causes the development of one erg of heat energy when the conductor is traversed by an absolute unit of current for one second.

The practical unit of resistance is 10^9 times as large as the absolute and is called the *Ohm*.

The *International Ohm* is the resistance of a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a uniform cross-sectional area and of a length of 106.300 centimeters.

(*g*) The *Absolute Electromagnetic Unit of Electromotive Force* is the electromotive force developed in a single conductor by a change of magnetic flux normal to the plane of the conductor at the rate of one maxwell per second.

The practical unit of electromotive force is 10^8 absolute units and is called the *Volt*.

The *International Volt* is that electromotive force which when applied to a conductor having a resistance of one international ohm produces a current of one international ampere.

(*h*) The *Absolute Electromagnetic Unit of Capacity* is the electric capacity of a system which stores up one electromagnetic unit of charge when one electromagnetic unit of electromotive force is impressed on the system.

The practical unit of capacity is 10^{-9} times as large as the absolute unit and is called the *Farad.*

The *International Farad* is the electric capacity of a system that receives a charge of one international coulomb when impressed by an electromotive force of one international volt.

(*i*) The *Absolute Electromagnetic Unit of Inductance* is the inductance of an electric circuit in which one absolute unit of electromotive force is developed by a change of current strength in the circuit at the rate of one absolute unit of current per second.

The practical unit of inductance is 10^9 times as great as the absolute unit and is called the *Henry.*

The *International Henry* is the inductance of an electric circuit in which one international volt is developed by a change of current strength in the circuit at the rate of one international ampere per second.

33. The Moving Coil Galvanometer. — The d'Arsonval or moving coil galvanometer consists of a coil of wire suspended between the poles of a permanent magnet. Its action depends upon the torque produced by the interaction of the magnetic field due to the current-carrying coil and that due to the permanent magnet. The value of this torque will now be derived.

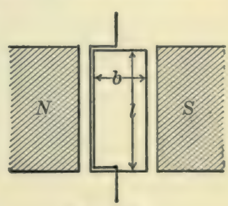

FIG. 55.

Consider a rectangular coil of n turns of length l cm. and breadth b cm. suspended in a uniform magnetic field of strength H gausses. If the coil carry i units of current, then the force on one side of the coil due to a single turn of wire will be (Art. 32)

$$F = ilH \text{ dynes.}$$

When the plane of the coil is in the plane of the field of the magnet the torque due to this force on one turn will be

$$L\left[= F\frac{b}{2}\right] = \frac{ilHb}{2}.$$

As the current flows in the opposite direction in the other side of the coil, the torque produced on both sides of the coil will be in the same direction. Hence, the total torque acting on the n turns is

$$L = nilHb \text{ dyne-cms.}$$

On substituting for the area lb, the symbol A, we have

$$L = niHA. \tag{151}$$

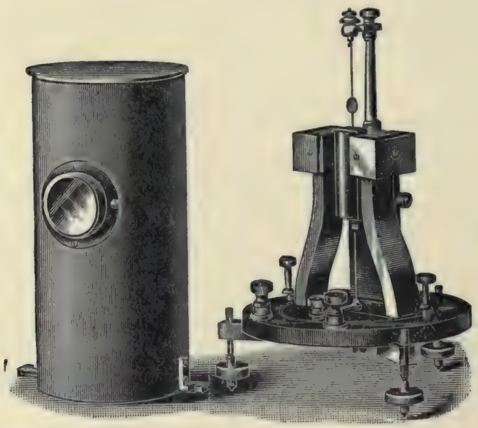

FIG. 56.

This is the value of the torque so long as the field of force due to the magnet is in the plane of the coil. In order that this condition may be maintained as the coil turns, the field of force due to the magnet must be radial. This can be accomplished by means of pole faces of the proper shape, together with a fixed soft iron core inside the coil.

The torque acting on the coil due to the interaction of the two magnetic fields is counteracted by a torque developed either by the twisting of a very thin straight metal ribbon as illustrated in Fig. 56, or by that developed by the coiling of a flat hair spring as shown

in Fig. 57. The former construction is used in sensitive stationary laboratory instruments, and the latter in portable instruments. In each case the current is led into and out of the coil by means of these metal strips.

If the torque developed by the suspending ribbon or spring varies directly with the angular deflection, and the field due to the permanent magnet is radial and of constant magnitude, then the deflection of the coil from the equilibrium position will be directly proportional to the current traversing the coil. Some direct reading instruments have such a uniform scale. In most instruments, however, no attempt is made to meet these conditions and the scale is ununiform, that is,

FIG. 57.

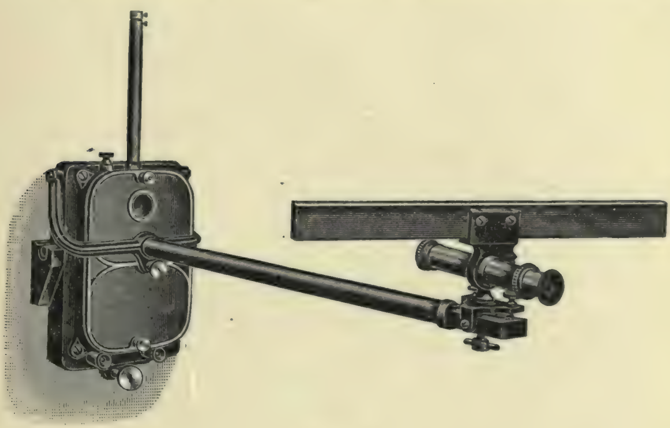

FIG. 58.

at different parts of the scale one degree of deflection does not correspond to the same change of current. Such instruments must be calibrated step by step throughout the range to be used.

The deflection of a portable galvanometer is indicated by a light pointer attached to the coil and moving over a circular scale. For sensitive galvanometers a ray of light serves as a pointer. In the latter case, a mirror attached to the moving system reflects light either from a horizontal scale into an observing telescope, Fig. 58, or from a linear light source to a translucent scale.

34. Damping. — Unless measures are taken to prevent it, the suspended system of a galvanometer when joined to a source of electromotive force will not come immediately to a steady deflection, but will oscillate back and forth through the position of equilibrium with gradually diminishing amplitude of swing. To permit quick readings, these oscillations must be damped. The damping may be produced mechanically by a light vane V attached to the moving system, Fig. 59, which, in plowing through the air, quickly reduces the amplitude of the oscillations. It may also be produced electromagnetically through the action of eddy currents set up in a closed loop of metal attached to the moving coil. When the coil rotates in the magnetic field produced by the galvanometer magnet, an electromotive force is set up in the loop of metal. This develops a current in the direction that will oppose the motion that produces it (Lenz' Law).

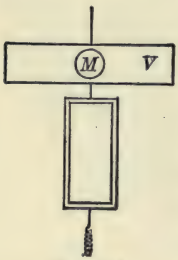

FIG. 59.

35. The Ballistic Galvanometer. — If instead of a steady current, an electric discharge be sent through a galvanometer, the suspended system will give a sudden kick or throw and then swing back and forth through the equilibrium position with decreasing amplitudes till the energy of motion is absorbed by damping. If the suspended system have such a great moment of inertia that it is not appreciably deflected from the equilibrium position before the discharge is concluded and if the throw be not too great, the throw will be directly proportional to the quantity of discharge. In this case,

$$Q = Gd, \tag{152}$$

where the constant of proportionality G is known as the "ballistic constant."

A galvanometer having a suspended system of such great moment of inertia that the throws produced are proportional to the quantity discharged through it is called a *ballistic galvanometer*.

When a galvanometer coil forming part of a closed circuit is swinging in a magnetic field, there will be induced in the coil currents in such directions as to oppose the motion and so reduce the throw. Whereas when the circuit is open, as when the galvanometer is connected to the plates of a condenser, the current set up in the coil by electromagnetic induction will be zero, and there will be no electromagnetic damping. It is thus seen that the ballistic constant of a galvanometer depends upon the resistance of the circuit.

36. The Vibration Galvanometer. — If an alternating current be applied to a galvanometer, the suspended system will receive a series of impulses alternating in direction. If the frequency of the current be high, the suspended system will move in neither direction unless the moment of inertia is very small. By making the moment of inertia sufficiently small, however, the suspended system will swing back and forth with the frequency of the impressed current. The amplitude of swing depends upon the magnitude of the current and also upon the nearness with which the natural period of vibration of the suspended system agrees with that of the alternating current. When the periods are equal, the amplitude of swing for a given current will be maximum.

Fig. 60.

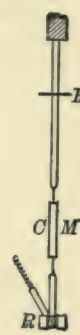

Fig. 61.

In Fig. 60 is represented the suspended system of a moving coil ballistic galvanometer, and in Fig. 61 that of a vibration galvanometer. The current enters and leaves the coil C of the latter by means of two stretched parallel wires. The natural period of the system is controlled by changing the tension of these wires and also by changing the length of the moving portion by adjusting the position of a fret F. The amplitude of swing of the suspended system is indicated by the image of a narrow slit of light. Light from a narrow slit after reflection from a tiny mirror M attached to the moving system traverses a

lens and forms an image of the slit. When the suspended system vibrates the image broadens out into a band of light. The vibrating system is then "tuned" by altering the length and tension of the suspension till the length of the band of light is maximum. This length is a measure of the alternating current. The vibration galvanometer is especially useful as an indicator of zero alternating current.

37. Series and Parallel Resistance. — Conductors arranged so that the same current traverses all of them, one after the other, are said to be in *series*. The resistance of several conductors in series equals the sum of the individual resistances. Thus

$$R = r_1 + r_2 + r_3 + \cdots .$$

Conductors so arranged that the current divides and a part traverses each branch are said to be in *parallel* or in *multiple arc.* In Fig. 62, let r_1, r_2, r_3, and r_4 be resistances in parallel. We shall now find the resultant resistance between a and b. Denoting the total current by I, and the current in the branches by i_1, i_2, i_3, i_4, respectively, we have

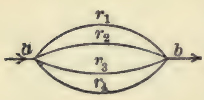

FIG. 62.

$$I = i_1 + i_2 + i_3 + i_4.$$

Substituting for each current its value as given by Ohm's law, we have:

$$\frac{V_{ab}}{R} = \frac{V_{ab}}{r_1} + \frac{V_{ab}}{r_2} + \frac{V_{ab}}{r_3} + \frac{V_{ab}}{r_4},$$

or,

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}. \tag{153}$$

38. Galvanometer Shunts. — In order that a galvanometer may be sensitive, the moving system must be light and the coil must have many turns of wire. When the wire is small, large currents cannot be transmitted by it. When large currents are to be measured, a shunt or by-pass is placed in parallel with the galvanometer coil which serves to transmit a large part of the current and leave only a small part to traverse the coil. If one knows the fraction of the total current that traverses the galvanometer, one

can obtain the value of the total current by taking the product of the galvanometer reading and this "multiplying power of the shunt."

The value of the multiplying power of the shunt will now be obtained. Represent the potential difference between a and b, Fig. 63, by the symbol V; the current in the line to be measured by I; the current in the galvanometer by i_g; the current in the shunt by i_s; the resistance of the galvanometer and that of the shunt by g and s, respectively. Then,

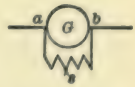

FIG. 63.

$$I = i_g + i_s,$$
$$i_g = \frac{V}{g}, \tag{154}$$
$$i_s = \frac{V}{s},$$

so that
$$I\ [= i_g + i_s] = V\left(\frac{s+g}{sg}\right). \tag{155}$$

Dividing each member of (155) by the corresponding member of (154),

$$I = i_g\left(\frac{s+g}{s}\right). \tag{156}$$

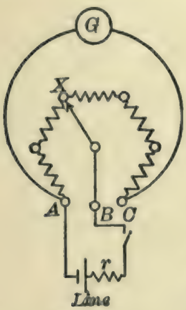

FIG. 64.

The quantity within the parenthesis is the multiplying power of the shunt.

39. The Universal Shunt. — The Ayrton universal shunt is a device to diminish the sensitivity of any galvanometer by a known amount. In Fig. 64 a high resistance AXC is connected to the galvanometer terminals A and C. One end of the line carrying the current to be measured is joined to A and the other end may be moved along the resistance AXC.

If the galvanometer resistance be represented by g, the resistance of AXC by R, that from A to X by s, and the line current by I',

then the galvanometer current when the line is joined to X will have the value, (156),

$$i_g' = I' \frac{s}{s + (R - s) + g} = I' \frac{s}{R + g}, \tag{157}$$

and when the line is joined to C, the galvanometer current

$$i_g'' = I'' \frac{R}{R + g}, \tag{158}$$

where I'' is the new line current.

From these two equations, we find

$$\frac{i_g'}{i_g''} = \frac{I's}{I''R}. \tag{159}$$

It is thus seen that the ratio of two galvanometer currents is independent of the galvanometer resistance.

If the resistance r of the line be large compared with that of the remainder of the circuit, I' will very nearly equal I'' and we may write

$$\frac{i_g'}{i_g''} \doteq \frac{s}{R}. \tag{160}$$

Usually the shunt is divided into coils ending at brass blocks so arranged that by moving either a contact arm, Fig. 65, or connecting plugs, the ratio s/R can be made 0.1, 0.01, or 0.001. Thus, depending upon the value of s/R employed, the deflection obtained, multiplied by 10, 100, or 1000, will give the deflection that would have been obtained if all the resistance R had been in the shunt.

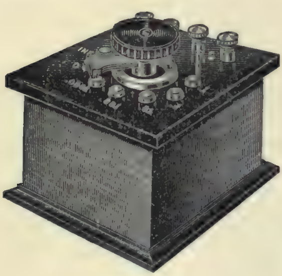

FIG. 65.

40. The Ammeter.—The ammeter is a low-resistance galvanometer provided with a scale graduated so as to indicate amperes directly. The resistance of an ammeter seldom exceeds a few thousandths of an ohm. Most ammeters are pivoted moving coil galvanometers provided with one or more shunts. The various shunts are made use of by connecting to different bind-

ing posts on the instrument. For each shunt there is a different multiplying factor. For example, the scale of a given instrument without a shunt in circuit may indicate amperes directly, while with one shunt the scale readings must be multiplied by 10, and with another shunt they must be multiplied by 100.

41. The Potential Galvanometer. The Voltmeter. — A high-resistance galvanometer may be connected to two points without sensibly changing the potential difference between them. Since the resistance of the instrument is constant, the deflection is not only proportional to the current traversing the instrument but also to the potential difference at the terminals of the instrument. A high-resistance galvanometer graduated so as to indicate potential difference directly in volts is called a *voltmeter*.

A voltmeter is made by adding to a milliammeter a series resistance of such value that the potential difference to be measured shall not be materially altered by the introduction of the instrument. The values of the scale divisions are obtained by taking the products of the resistance of the instrument and the values of current giving the various deflections. The relation of the potential difference between two points before the introduction of the instrument to the potential difference between the same points after the introduction of the instrument can be determined as follows: Denote the potential difference between the points a and b before being connected by the galvanometer by V and the current traversing the line by I. When a and b are connected by the galvanometer, the corresponding quantities will have values represented by V' and I', respectively. Denote the resistance of the conductor from a to b by r and the resistance of the galvanometer and accessories by g. Then,

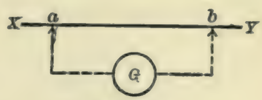

FIG. 66.

$$V = Ir \tag{161}$$

and

$$V' = I' \frac{rg}{r+g}. \tag{162}$$

Dividing each member of (161) by the corresponding member of (162)

$$\frac{V}{V'} = \frac{I(r+g)}{I'g}. \tag{163}$$

A value of the ratio of I to I' will now be found and substituted in this equation. If we represent by E the electromotive force of the circuit, and by R the resistance of the part of the circuit to the left of a and to the right of b, we may write

$$E = I\,(R + r) = I'\left(R + \frac{rg}{r+g}\right),$$

or,

$$\frac{I}{I'} = \frac{R\,(r+g) + rg}{(r+g)\,(R+r)}.$$

Substituting this value in (163)

$$\frac{V}{V'} = \frac{Rr}{(R+r)\,g} + 1. \qquad (164)$$

This equation shows that by increasing the resistance of the galvanometer and accessories g, the indicated potential difference V' approaches as a limit the required potential difference V. Ordinary voltmeters have a resistance of about 100 ohms for each volt of the scale. For example, a voltmeter with a range 0–150 volts would have a resistance of about 15,000 ohms.

42. The Voltmeter Multiplier. — Suppose it be required to measure a potential difference greater than a given voltmeter will indicate. It will now be shown that by putting in series with the potential galvanometer or voltmeter an extra resistance M, Fig. 67, the instrument can be used to measure a potential difference V' greater than its scale indicates.

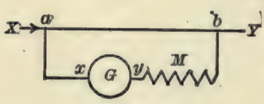

FIG. 67.

Let the resistance of the galvanometer be g, the current through it be i_g, the potential difference at its terminals be V_g, and the potential difference between a and b be V'. Then,

$$V' = i_g\,(g + M) \qquad (165)$$

and

$$V_g = i_g g. \qquad (166)$$

Dividing each member of (165) by the corresponding member of (166), the value of the potential difference between a and b when the galvanometer is in place is seen to be

$$V' = V_g\left(\frac{g+M}{g}\right). \qquad (167)$$

The device of resistance M placed in series with a voltmeter for the purpose of increasing the range of the instrument is called a *voltmeter multiplier.* The fraction within the parenthesis of (167) is called the multiplying power of the voltmeter multiplier. If the multiplying power of the voltmeter multiplier is to have some assigned value n, that is if

$$\frac{g + M}{g} = n,$$

it follows that the resistance of the multiplier must have the value

$$M = (n - 1)\,g. \tag{168}$$

For example, if it be desired that a given voltmeter be made available for measuring potential differences ten times as great as its scale indicates, the voltmeter would be connected in series with a resistance nine times as great as the resistance of the voltmeter.

Voltmeters are often provided with two scales, one to be read when a multiplier is in circuit, and the other when the multiplier is not in circuit.

43. The Sensitivity of a Galvanometer. — The sensitivity of a galvanometer is expressed in three ways:

(*a*) The number of amperes necessary to cause a deflection of one millimeter on a scale distant one meter. This is commonly called the Figure of Merit.

(*b*) The number of ohms or megohms * that must be placed in series with the galvanometer in order that an impressed electromotive force of one volt shall cause a deflection of one millimeter on a scale distant one meter.

(*c*) The number of volts necessary to cause a deflection of one millimeter on a scale distant one meter.

It will be noticed that in each case the galvanometer is traversed by the current represented by the figure of merit. If the figure of merit be known, the sensitivity expressed in either of the other ways is readily obtained. Thus, if the figure of merit be repre-

* Meg = million.

sented by F, the sensitivity expressed in ohms for unit deflection with one volt will be

$$R'\left[=\frac{E}{I}\right]=\frac{1}{F}$$

and the sensitivity expressed in megohms is

$$R''=\frac{1}{1{,}000{,}000\,F}. \tag{169}$$

Again, the number of volts necessary for unit deflection on a scale distant one meter is

$$E\,[=IR]=Fg, \tag{170}$$

where g is the resistance of the galvanometer.

44. Standards of Resistance. — The material employed for standard resistance coils should have a high resistivity that does not change with time or use, and have a low temperature-resistance coefficient and low thermoelectric power. An alloy called manganin consisting of 0.84 copper, 0.12 manganese, and 0.04 nickel fulfils best these requirements.

A coil of many turns wound like a spool of thread would have such a large self-inductance and capacity that it would give erroneous results when used with variable currents. The self-inductance can be made negligible by so winding the coil that the direction of the current in adjacent turns is opposite, and the capacity can be reduced by so winding that there is only a small potential difference between adjacent turns. The capacity effect is usually of much less importance than the inductive effect and its elimination is usually not attempted.

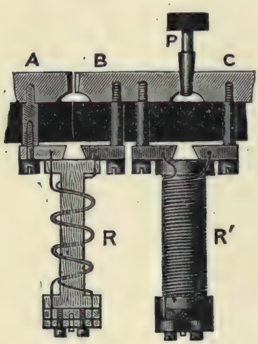

Fig. 68.

The simplest method of avoiding self-induction is by the bifilar method of winding which consists in doubling the wire in the middle and winding the two parts side by side as shown in Fig. 68. As there is a considerable potential

difference between all adjacent turns except near the end of the doubled wire, a coil of many turns wound in this manner has appreciable capacity.

To reduce the capacity as well as self-inductance, the windings must be so arranged that the current in adjacent turns is not only in opposite directions but that, in addition, the potential difference between adjacent turns shall be small. A solution of this problem is furnished by the Curtis winding illustrated in Fig. 69.

FIG. 69.

FIG. 70.

The coil is wound on a split tube of porcelain, and after each turn the wire is carried through the slit and wound in the reverse direction.

Standard resistance coils are usually mounted in metal cases arranged so that they can be immersed in an oil bath of known temperature. One form of mounting used for single coils is shown in Fig. 70.

45. Resistance Boxes. — Sets of coils arranged so that various known resistances may be readily obtained are commonly mounted in wooden boxes. A common form of resistance box is illustrated in Fig. 71. In this form, the terminals of each coil are soldered to massive brass blocks on the vulcanite top of the box. The gaps between adjacent blocks can be filled by tapered brass plugs as shown in Fig. 68. The resistance between the binding posts equals the sum of the resistances between the ends of the unplugged gaps.

Resistance boxes are also in common use in which the coils are put into circuit or taken out of circuit by means of dial switches instead of by plugs. The top of a box with dial switches is illustrated in Fig. 75.

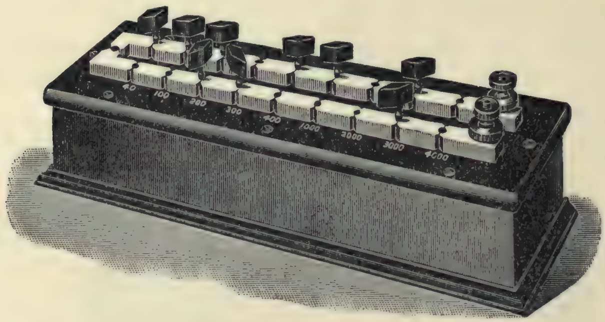

Fig. 71.

46. The Wheatstone Bridge. — Consider two conductors ABC and ADC, Fig. 72, joined in parallel to the terminals of a battery. Corresponding to any point B on the conductor ABC, there is a point D on the conductor ADC which is at the same potential. If two such points be joined by a conductor BD, no current will flow along this bridge wire Represent the potentials at the points A, B, C, and D, by the symbols V_A, V_B, V_C, and V_D, respectively. Let the resistances of the arms AB, AD, BC, and DC be denoted by R_1, R_2, R_3, and R_4, respectively. Let the current intensities in these arms be i_1, i_2, i_3, and i_4, respectively. In the case considered, since $V_B = V_D$, $i_1 = i_3$, and $i_2 = i_4$. Then by Ohm's Law,

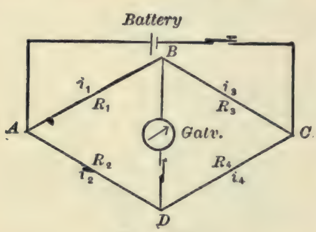

Fig. 72.

$$i_1 = \frac{V_A - V_B}{R_1}, \qquad i_3\,[= i_1] = \frac{V_B - V_C}{R_3},$$

$$i_2 = \frac{V_A - V_B}{R_2}, \qquad i_4\,[= i_2] = \frac{V_B - V_C}{R_4}.$$

Whence,
$$\frac{V_A - V_B}{R_1} = \frac{V_B - V_C}{R_3}, \tag{171}$$
$$\frac{V_A - V_B}{R_2} = \frac{V_B - V_C}{R_4}. \tag{172}$$

Dividing each member of (171) by the corresponding member of (172) we have
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \tag{173}$$

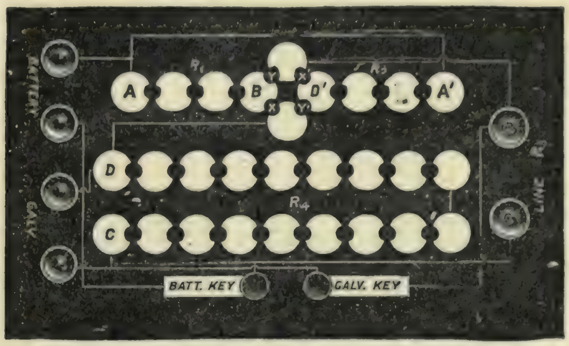

FIG. 73.

Thus, if any three of the resistances are known when no current flows through the bridge wire, the remaining unknown resistance can be determined. A galvanometer, G, indicates the presence or absence of current in the bridge wire.

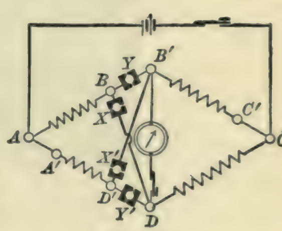

FIG. 74.

For general resistance measurements, a Wheatstone bridge having three groups of coils of known resistance, with convenient arrangements for altering the resistance of each group is usually employed. For laboratory use, the box containing the coils is separate from the galvanometer and battery. For shop use when portability is essential, the coils, the galvanometer, and battery are enclosed in a single box. In the arrangement of the tops of the boxes of coils there is great variety. In some,

the resistances are changed by means of plugs, and in others by means of sliding switches. The top of a common form of cheap box of coils is shown in Fig. 73. The lettering on this plan corresponds with that in Figs. 72 and 74.

In the model shown in Figs. 73 and 74, when there is balance with the gaps Y and Y' filled by plugs and the gaps X and X' open, we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4},$$

whereas, with the gaps Y and Y' open and the gaps X and X' filled by plugs, the resistances R_1 and R_2 will be interchanged, so that

$$\frac{R_2}{R_1} = \frac{R_3}{R_4}.$$

By this switching device the box requires two less coils than otherwise would be needed.

The series of coils AB, AD, BC, and DC comprising the resistances R_1, R_2, R_3, and R_4, respectively, constitute the "arms" of the

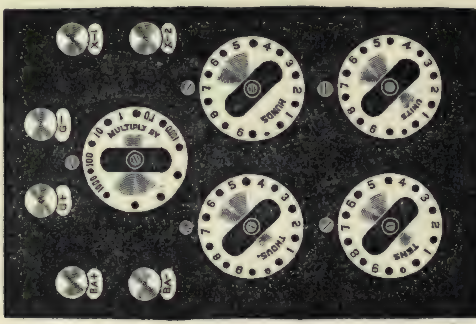

FIG. 75.

Wheatstone bridge. The arms AB and AD are called the "ratio arms," and the other arm in the box is called the "rheostat arm." The fourth arm is the resistance being measured.

The plan of a box top in which rotating switches are used instead of plugs is shown in Fig. 75.

47. The Slide Wire Wheatstone Bridge. — The Wheatstone bridge principle can be applied so that only one known resistance is required. Suppose that the conductor ADC, Figs. 76 and 77, is a long uniform straight wire. Let l_2 and l_4 be the lengths of this wire from A to D and from D to C, respectively. Then since the

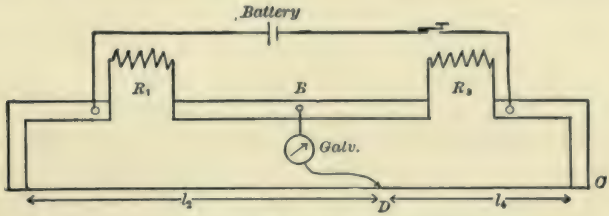

FIG. 76.

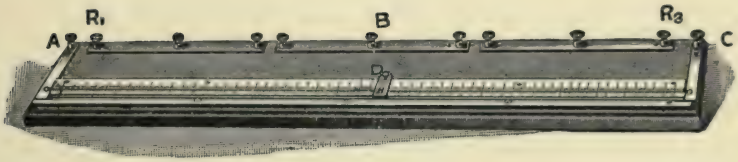

FIG. 77.

resistance of a uniform wire is proportional to its length, it follows that when no current passes along the bridge BD,

$$\frac{R_1}{R_3}\left[=\frac{R_2}{R_4}\right]=\frac{l_2}{l_4}. \qquad (174)$$

Then, knowing either the resistance R_1 or R_3 together with the lengths l_2 and l_4, the remaining resistance can be determined.

The end D of the bridge wire is moved back and forth along the straight wire till the galvanometer indicates no current. The position of the sliding contact D is indicated by a meter scale, Fig. 77, directly below the wire. This particular form of Wheatstone bridge is called the "slide wire bridge" or "meter bridge."

The adjustment of the slide wire bridge is most sensitive when the contact D is at the middle of the slide wire at the time no current flows through the galvanometer. This means that the

known resistance should be nearly equal to the unknown resistance. For this reason a slide wire bridge designed to measure resistances extending through a considerable range must be provided with a considerable number of coils of known resistances.

48. Slide Wire Bridge Errors. — The slide wire bridge is subject to two errors. One possible source of error is due to the resistance of the end connection from A to R_1, Fig. 76, being different than that of the end connection from C to R_3. Another error results if the index of the slide D be not exactly over the contact edge. These two errors can be reduced to negligible dimensions by the procedure now to be explained.

If with $R_1 \doteq R_3$ and $l_2 \doteq l_4$, the bridge is in balance, the difference of the end resistances is negligible compared with the resistance of l_2 or l_4. Denoting by d the "tapping error," that is, the error due to the index of the slider being not directly above the contact edge, we then have

$$\frac{R_1}{R_3} \doteq \frac{l_2 - d}{l_4 + d}. \tag{175}$$

If the resistance coils R_1 and R_2 be interchanged,

$$\frac{R_1}{R_3} \doteq \frac{l_4' + d}{l_2' - d}, \tag{176}$$

or,

$$\frac{l_2 - d}{l_4 + d} \doteq \frac{l_4' + d}{l_2' - d}\left[\doteq \frac{R_1}{R_3}\right].$$

By composition,

$$\frac{(l_2 - d) + (l_4' + d)}{(l_4 + d) + (l_2' - d)} \doteq \frac{l_2 - d}{l_4 + d}\left[\doteq \frac{R_1}{R_3}\right].$$

Whence,

$$\frac{l_2 + l_4'}{l_2' + l_4} \doteq \frac{R_1}{R_3}. \tag{177}$$

Denoting the entire length of the slide wire by L, where $L = l_2 + l_4 = l_2' + l_4'$, we may write (177) in the form

$$\frac{R_1}{R_3} \doteq \frac{L - l_2' + l_2}{L - l_2 + l_2'} \doteq \frac{L - (l_2' - l_2)}{L + (l_2' - l_2)}. \tag{178}$$

That is, if R_1 is nearly equal to R_3 the end error and the tapping error can be made negligible by substituting in (178) the scale

reading l_2 when the bridge is in balance, and the new scale reading l_2' obtained by interchanging R_1 and R_3 and again balancing the bridge.

49. Resistivity. — From experiment it is found that the resistance of a conductor of length l and area of cross section A is expressed by the equation

$$R = \frac{\rho l}{A},$$

where ρ is a constant for any given material at any given temperature. This quantity is called the *resistivity* of the given material at the given temperature.

If resistance be expressed in microhms,* and length and radius in centimeters, then the resistivity of a wire

$$\rho = \frac{R\pi r^2}{l} \text{ microhms per centimeter cube.} \tag{179}$$

When the length of a wire is expressed in feet, the diameter is often expressed in thousandths of an inch. A thousandth of an inch is called a *mil*, and the area of a circle one mil in diameter is called a *circular mil*. The area of a circle d mils in diameter is d^2 circular mils. Thus, if length be expressed in feet, diameter in mils, and resistance in microhms, the resistivity of a wire of circular cross section

$$\rho' = \frac{Rd^2}{l'} \text{ microhms per circular mil foot.} \tag{180}$$

50. Electrolytic Resistance. — In measuring the resistance of electrolytes a modification of Wheatstone's bridge is employed. Direct current must not be passed through an electrolyte on account of the excessive polarization and consequent back electromotive force, increase of resistance and electrostatic capacity that would be produced. Polarization is reduced by the use of a symmetrical alternating current of a frequency not less than 1000 cycles per second. When an alternating current is used, a sensitive telephone is substituted for the galvanometer usually employed in the Wheatstone bridge.

* Micro = millionth.

Even with an alternating current there is some polarization. Polarization acts as an added resistance and as a condensive reluctance. The apparent increase in resistance arises from the incomplete reversal of the electrode reaction. This is diminished by the use of high frequencies and large platinized electrodes. Capacity at the electrodes gives rise to an unbalance of the Wheatstone bridge even though the resistances are in balance. When there is an unbalanced capacity a setting can be made which will give a minimum of sound in the telephone, but the center of the minimum does not represent the correct bridge setting. To obtain complete silence in the telephone it is necessary that: (*a*) the polarization at the electrodes be equal and symmetrical, (*b*) the condensive reluctance of the cell be either negligible or balanced by inductance or capacity in the remainder of the circuit, (*c*) the fundamental note in the telephone be unaccompanied by overtones.

Conditions (*a*) and (*c*) require that the impressed alternate current wave shall be of a true sine wave form. High-frequency currents of true sine wave form can be obtained from a Vreeland oscillator or from a specially designed dynamo. Induction coils give intermittent currents that depart so much from the sine wave form that they can be used only when approximate results are sufficient.

To sum up, accurate determinations of electrolytic resistance may be made by the Wheatstone bridge method using as a source of electromotive force a high-frequency generator giving a pure sine wave of constant frequency, using as a detector a telephone tuned to the frequency employed and using standard resistances free of capacity and inductance.

The cells for holding the electrolyte are of various shapes depending upon the magnitude of the resistance to be measured. A few modern patterns are illustrated in Figs. 78–80.

To find directly the resistivity of a given electrolyte, the length and area of cross section of the conducting column of liquid must be known. But after the resistivity of one electrolyte has been determined directly, the resistivity of any other electrolyte can be easily obtained from a comparison of the resistances of a given cell

of any shape first when filled with one electrolyte and then with the other. Thus, representing the resistances of a cell when filled with the standard electrolyte and when filled with the electrolyte

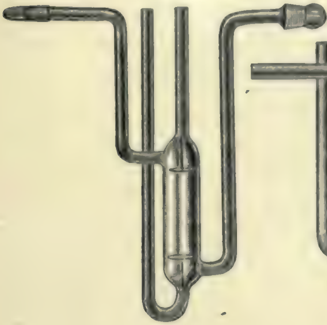
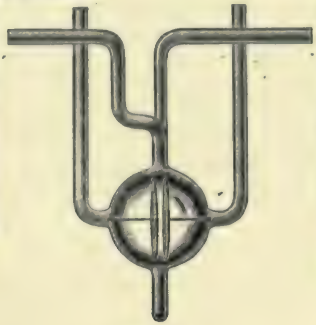
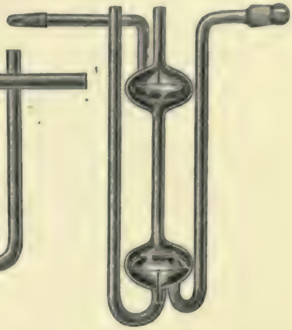

FIG. 78. FIG. 79. FIG. 80.

under test by R_s and R_x, respectively, and the resistivities by ρ_s and ρ_x, we have

$$R_s = \rho_s \frac{l}{A} \tag{181}$$

and

$$R_x = \rho_x \frac{l}{A}, \tag{182}$$

where l and A represent the length and area of the column of conducting liquid.

Dividing each member of (182) by the corresponding member of (181), we have

$$\rho_x = \rho_s \frac{R_x}{R_s}. \tag{183}$$

Since l and A do not appear in this equation, this method of comparing resistivities requires no knowledge of the dimensions of the cell.

Since conductivities are inversely proportional to resistivities, we have from the above equation,

$$k_x = k_s \frac{R_s}{R_x}, \tag{184}$$

where the conductivities of the standard solution and the solution under test are represented by k_s and k_x, respectively.

51. Standard Cells. — For a laboratory standard of electromotive force either the Clark cell or the Weston cell is employed.

The Clark standard cell is a cell having for the positive electrode pure mercury covered with a paste consisting of pure mercurous sulphate mixed with finely divided mercury and pure zinc sulphate crystals and solution, and for the negative electrode an amalgam containing ten per cent by weight of pure zinc covered with a layer of pure zinc sulphate crystals, the electrolyte being a saturated solution of pure zinc sulphate. At 20° C., this cell has an electromotive force of 1.434 volts. At any other temperature t not far from 20 degrees, the electromotive force is

$$E_t = 1.434 - 0.00115\,(t - 20) \text{ volts.} \qquad (185)$$

The Weston standard cell is similar to the Clark cell except that instead of zinc sulphate is employed cadmium sulphate, and instead of the zinc amalgam there is employed an amalgam containing 12.5 per cent by weight of pure cadmium. At 20° C., the electromotive force of this cell is 1.0183 volts. At any other temperature t differing not much from 20° the electromotive force is

$$E_t' = 1.0183 - 0.00004\,(t - 20) \text{ volts.} \qquad (186)$$

The above values are obtained when all the materials entering into the construction of the cell are as pure as it is known how to produce. Substances known in the chemical trade as "C. P." will not suffice. Impurities of the mercurous sulphate cause the greatest departure. But with properly retreated C. P. mercurous sulphate and the other materials of the C. P. grade, a cell will differ in electromotive force only about 0.01 per cent from those made of the most carefully prepared materials.

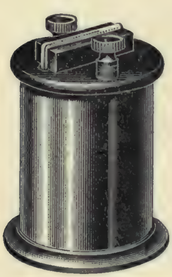

Fig. 81

If more than 0.0001 ampere be taken from a standard cell the electromotive force will drop appreciably. If a standard cell be short-circuited, it will be ruined. It is, therefore, of prime importance that a high resistance be always in circuit and

that an inappreciable current be taken from the cell for only very brief periods.

Standard cells are often mounted in cases with a thermometer as shown in Fig. 81.

52. The Potentiometer Method of Comparing Potential Differences. — Electromotive forces and potential differences can be determined by comparison with a standard cell. Consider a circuit containing a constant battery, an adjustable rheostat Rh, and a uniform homogeneous wire PP'. To one point V of this uniform wire let there be joined one end of a line containing a standard cell with its high resistance, and a galvanometer G and tapping key K. If the fall of potential along PP' be greater than the electromotive force of the standard cell, and if similar poles of the working battery and standard cell be joined to V, there will be some point on PP' which will be at the same potential as the other terminal V' of the standard cell. If V' be brought into contact with this point, no current will flow in the standard cell because the potential difference between V and V' due to the working battery and that due to the standard cell are equal and opposing one another. Now when a cell is giving zero current the potential difference at its terminals equals the electromotive force. Whence the potential difference between V and V' equals the known electromotive force of the standard cell.

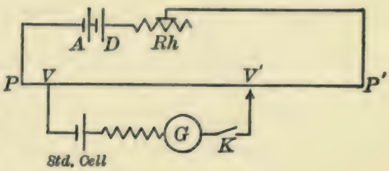

FIG. 82.

Since the wire PP' is uniform, the potential difference between any two points is directly proportional to the length of wire between them. Thus, after the potential difference between two points V and V' has been balanced against the standard cell, the potential difference between any other two points of the wire is known — so long as the electromotive force of the working battery does not change. Consequently, we can measure any potential difference, within the range of the slide wire, by substituting it for the standard cell and moving the sliding contact V' along the wire till the galvanometer G gives zero deflection.

It is convenient to have unit length of the slide wire represent one millivolt. This can be arranged as follows. Place the slider V' at the scale division that is numerically equal to the electromotive force of the standard cell. Thus, if a Weston standard cell of electromotive force 1.0183 volts be employed, and the millimeter be taken as the smallest scale division, the slider would be placed 1018.3 millimeters from V. Now adjust the rheostat till the galvanometer gives zero deflection on pressing the tap key K. So long as the current in the slide wire does not change, the millimeter scale under the slide wire will now indicate the potential difference in millivolts, between V and any other point of the wire.

By substituting for the standard cell the source under investigation, and moving the slider till a balance is again obtained, the required potential difference will be indicated on the scale.

The potential method has certain advantages over the voltmeter method. Thus, since no current is taken from the source during the moment of observation, (*a*) the potential difference is not altered by the introduction of the measuring device, (*b*) the indication is independent of the resistance of the lead wires, (*c*) the observed potential difference at the terminals of a source of electromotive force equals the electromotive force. Being a zero method, (*d*) variations in the sensitivity of the galvanometer introduced no error in the result. On the other hand, the standard cell must have a known value. This value will remain constant for years if the cell is never allowed to give an excessive current.

53. The Potentiometer. — The potentiometer principle furnishes highly accurate methods for the comparison of potential differences, resistances, and currents. In the comparison of the resistance of two coils, the two coils are connected in series with a source of constant current and the potential differences at the terminals of the coils measured. The ratio of the two resistances is then equal to the ratio of the potential differences at their terminals. In measuring a current, it is passed through a circuit containing a standard resistance coil, and the potential difference at the terminals of the coil measured. The current is then computed by Ohm's law.

A self-contained apparatus arranged for the ready application

of the potentiometer principle is called a potentiometer. The simplest potentiometer consists of a 150-cm. wire, PP', Fig. 83, adjustable resistance R, working battery, standard cell S, pole changer U, and sliding contact V or V_1.

Any potentiometer must meet the following conditions. (*a*) The resistance of the conductor PP' must be constant. (*b*) The

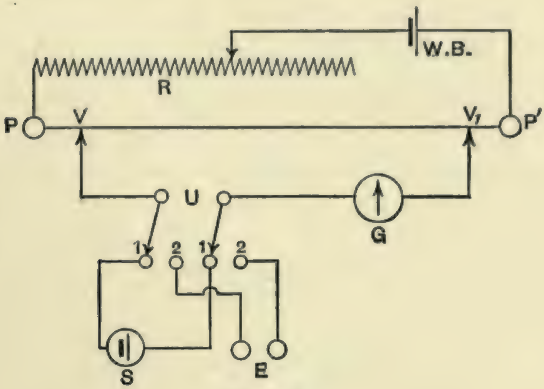

FIG. 83.

working current must be constant and have such a value compared with the resistance of PP' that the potential difference at the terminals P and P' is greater than the electromotive force of the standard cell. (*c*) The terminal of the standard cell and that of the working battery connected to V must be of the same sign. For convenience of manipulation, different designers have arranged the essential parts of the circuit in different ways. In some types, the potential difference between the points V and V_1 is varied, not by sliding V or V_1 along a uniform wire, but by increasing or decreasing the resistance between these points by throwing in or out additional coils of wire. In order that the resistance between P and P' may remain constant, an equal change of resistance in the opposite sense must be simultaneously made in the part between P and V or between V_1 and P'.

This arrangement is employed in the Wolff potentiometer now in extensive use. It is shown in diagram in Fig. 84, and in plan in

Fig. 85. In Figs. 83, 84, and 85, P, P', V, and V_1 refer to corresponding points. In order that the potential difference between V and V_1 may be varied by small steps, the conductor PP' consists

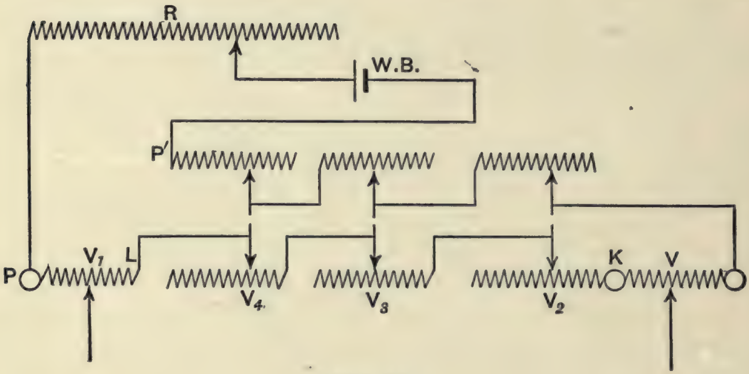

Fig. 84.

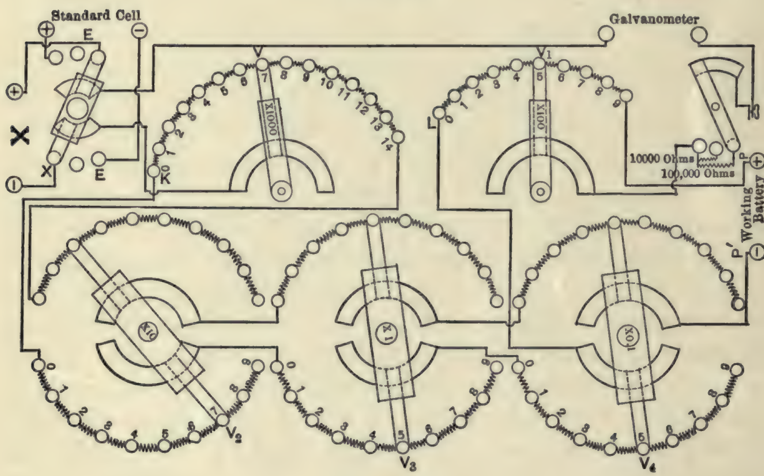

Fig. 85.

of a wire of rather large resistance arranged in groups of coils as indicated in the lowest line of Fig. 84. As the total resistance of the conductor PP' must be constant, a duplicate set of coils is provided for three of the five groups. These are shown to the right

of the letter P' in Fig. 84, and to the left of P' in Fig. 85. Moving along the groups of coils marked V_2, V_3, and V_4, and the corresponding compensating coils are sliding contacts mechanically connected but electrically insulated from one another, and so arranged that when a pair of connected contacts is moved so as to increase the resistance between V and V_1, an equal resistance is added to the remainder of the line PP'. In the upper left-hand corner of Fig. 85 is shown a switch by means of which either the standard cell or the potential to be measured can be put across VV_1. In the upper right-hand corner is another switch which is employed to put a high resistance in the galvanometer circuit until a balance is nearly attained.

In using this potentiometer a standard cell is placed in the circuit and the dials set to read a decimal multiple of its electromotive force. For example, if the cell has an electromotive force of 1.08130 volts, the resistance of the portion of the circuit between its poles would be made 10813.0 ohms. The working battery is then adjusted till there is no galvanometer deflection. The line current is now of such a value as to cause a potential drop of one volt across 10,000 ohms, that is, the current is 0.0001 ampere. By substituting for the standard cell a source of unknown potential difference and adjusting the sliding contacts till the galvanometer deflection is zero, the required potential difference is numerically equal to one-tenthousandth of the sum of the resistances indicated by the dials V, V_1, V_2, V_3, and V_4.

54. The Pyrovolter Method. — In this method the potential difference of the source under investigation is first balanced against the potential difference at the terminals of a known resistance traversed by a current, and then this current is measured by a galvanometer. The product of the measured current and the constant resistance equals the required potential difference. The circuit is represented in Fig. 86. With the dial switch

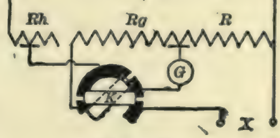

FIG. 86.

K in the position indicated, the main current goes from Ba through the rheostat Rh, the left-hand side of the switch, R_g, R and back to

the battery. By joining to the binding posts X, in the proper direction, the source whose potential difference is required, the potential difference at the terminals of the source is opposed by that at the terminals of R. The current in R can be adjusted by means of the rheostat till these two potential differences are equal. When balanced, the galvanometer gives zero deflection.

If the dial switch be now rotated into the dotted position, the current from the battery traverses Rh, the right-hand part of the switch, the galvanometer and R, back to the battery. That is, the current traverses the same circuit as before except that G takes the place of R_g. If the resistance of G equals that of R_g, the current through R is the same as before. That is, the current through R when the two potential differences were balanced is now indicated by the galvanometer. The product of the constant resistance R and the current producing any selected deflection can be marked beside the selected scale division, and thus the instrument divided so as to indicate potential differences directly.

The precision of the pyrovolter method is limited by that of the galvanometer. But the method is superior to the voltmeter method in that the potential difference being measured is not altered by the introduction of the device; the indication is independent of the resistance of the lead wires; and the observed potential difference at the terminals of a source of electromotive force equals the electromotive force.

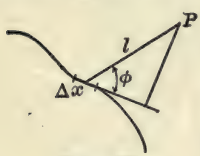

FIG. 87.

55. The Magnetic Field Strength at any Point on the Axis of a Circular Current-carrying Conductor. — Representing the length of a short element of current-carrying conductor by Δx, the distance from the center of this element to a given point P by l, the angle between l and Δx by ϕ, Ampere found by experiment that the strength of field at P due to a current I in the element Δx is

$$\Delta H = \frac{I\Delta x}{l^2} \sin \phi. \tag{187}$$

Ampere also found that the line of force at any point due to an element of a current-carrying conductor is the arc of a circle having the element as its axis, the plane of the circle being normal to the element.

It is now required to find the field strength at any point on the axis of a circular current-carrying conductor. Let *abc* represent

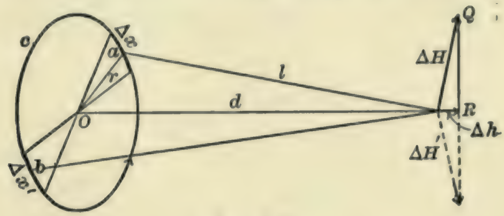

Fig. 88.

the circular conductor of radius r. Let the distance of the assigned point from the plane of the conductor be d. Consider an element of the conductor of length Δx. Let its distance from P be denoted by l.

Since l is perpendicular to Δx, the field strength at P due to a current I in Δx is (187)

$$\Delta H = \frac{I\Delta x}{l^2}. \tag{188}$$

The direction of this force is perpendicular both to Δx and r.

In a similar manner, find the field strength $\Delta H'$ at the point P, due to an element $\Delta x'$ diametrically opposite to Δx. The component of $\Delta H'$ perpendicular to the axis of the circular conductor is equal and opposite to the component of ΔH perpendicular to the axis. Consequently, the resultant force at any point on the axis of a circular current-carrying conductor equals the sum of the components, in the direction of the axis, of the force due to the separate elements into which the conductor has been considered to be divided.

Since the triangles OaP and PQR are similar, we have for the component of ΔH in the direction of the axis,

$$\Delta h = \frac{r\Delta H}{l}.$$

Substituting for ΔH its value in (188),

$$\Delta h = \frac{Ir\Delta x}{l^3}.$$

For every other element of the circular current we can obtain a similar expression for the intensity of field along the axis, and by summing all these expressions we get the total field at the point P due to the current in the circular conductor. Calling this total field H, we obtain

$$H = \frac{Ir \cdot 2\pi r}{l^3}.$$

Or, since

$$l = (r^2 + d^2)^{\frac{1}{2}},$$

$$H = \frac{Ir^2 \cdot 2\pi}{(r^2 + d^2)^{\frac{3}{2}}}.$$

If, instead of a single turn, there are n turns of wire in the circular conductor, then the intensity of field is

$$H = \frac{2\pi n r^2 I}{(r^2 + d^2)^{\frac{3}{2}}}. \qquad (189)$$

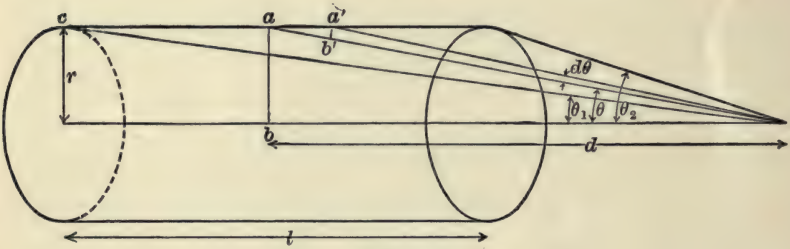

FIG. 89.

56. The Magnetic Field Inside a Solenoid. — Consider a cylindrical solenoid of length l, consisting of n' turns of wire per unit length, of average radius r. Let x'' be the average distance between the centers of wires in adjacent turns. The intensity of the magnetic field at P, Fig. 89, due to n'' turns of wire carrying current I, occupying the very short distance aa' measured along the axis of the solenoid is (189)

$$H_1 = \frac{2\pi r^2 n'' I}{(r^2 + d^2)^{\frac{3}{2}}},$$

or, since $aa' = n''x''$,

$$H_1 = \frac{2\pi r^2 I\,(aa')}{x''\,(r^2 + d^2)^{\frac{3}{2}}}. \tag{190}$$

This expression will now be put into a form containing only quantities that admit of accurate measurement. Draw $a'b'$ perpendicular to aP, and ab perpendicular to the axis of the solenoid.

Since the triangles $aa'b'$ and aPb are similar,

$$(aa') = (a'b')\,\frac{aP}{ab} = (a'b')\,\frac{aP}{r}.$$

And since

$$(a'b') = (a'P)\sin d\theta \doteq (a'P)\,d\theta \text{ [since } d\theta \text{ is very small]}$$
$$\doteq (aP)\,d\theta \text{ [since } aa' \text{ is as small as we please]}$$

we have (190)

$$H_1 = \frac{2\pi r^2 I\,(aP)^2\,d\theta}{rx''\,(r^2 + d^2)^{\frac{3}{2}}} = \frac{2\pi r I\,(aP)^2\,d\theta}{x''\,(aP)^3} = \frac{2\pi\,(ab)\,I\,d\theta}{x''\,(aP)} = \frac{2\pi I}{x''}\sin\theta\,d\theta.$$

This is the intensity of the magnetic field at P due to the current I in the element aa' of the solenoid. The field due to the entire solenoid is then

$$H = \frac{2\pi I}{x''}\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta = \frac{2\pi I}{x''}(\cos\theta_1 - \cos\theta_2). \tag{191}$$

Representing the total number of turns of wire in one layer of the solenoid by n,

$$n' = \frac{n}{l} \quad \text{and} \quad n = \frac{l}{x''}.$$

Whence,

$$n' = \frac{1}{x''}.$$

Consequently, (191) becomes

$$H = 2\pi n' I\,(\cos\theta_1 + \cos\theta_2). \tag{192}$$

If the solenoid be very long compared to its radius, and the point P be taken near the center, then θ_1 and θ_2 will be so small that $\cos\theta_1$ and $\cos\theta_2$ will each become very nearly equal to unity.

Therefore at the center of a solenoid that is long compared to its radius, the magnetic field intensity is

$$H = 4\pi n'I \text{ gausses.} \tag{193}$$

If the average area of cross section of the turns is A, then the magnetic flux inside the solenoid is

$$\Phi\,[= AH] = 4\pi n'IA \text{ maxwells.} \tag{194}$$

57. Condensers. — A condenser consists of two conducting plates separated by a dielectric. For a condenser of large capacity the plates should have large area, be separated by a short distance, and the space between them filled with an insulating medium of high dielectric constant. The simplest form of condenser consists of two metal plates separated by air. But an air condenser has small capacity not only because air has a small dielectric constant, but also because the plates cannot be brought very near one another without one plate discharging through the air to the other. For condensers of large capacity, the dielectric is usually paraffined paper or mica.

One common form of paraffined paper condenser, much used in telephones, consists of two long strips of tinfoil separated by wider strips of paraffined paper, and the whole rolled into a compact cylindrical or flat form, Fig. 90. Mica condensers consist of piles of sheets of tinfoil separated by thin layers of mica. By joining the odd numbered sheets of tinfoil, and also the even numbered sheets, as in Fig. 91, we have the same result as though there were but two large sheets of tinfoil separated by two large sheets of mica.

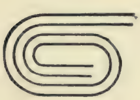

Fig. 90.

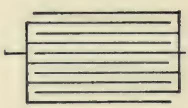

Fig. 91.

The potential difference between the plates of a charged paraffined paper condenser gradually diminishes for a time even when the insulation is perfect. And if such a condenser after being charged for some time be discharged, not all of the charge will disappear. By waiting for a while another discharge can be produced. The absorption and residual charge are practically zero for mica con-

densers. For this reason, mica condensers are used whenever accurate measurements depending upon constant capacities are to be made. Paraffin condensers can be used, however, when the greatest precision is not required if care be taken to charge quickly and discharge immediately. The duration of the time of charging should not exceed one-half second.

The ordinary form in which standard condensers are mounted is shown in Fig. 92. Each of the horizontal bars is joined to one

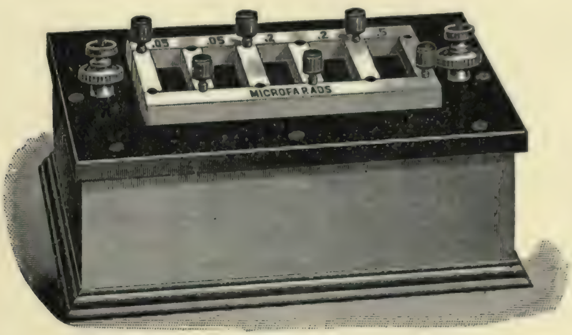

FIG. 92.

of the binding posts. To the first and second cross bars are connected the plates of a condenser of 0.05 microfarad capacity. To the second and third cross bars are connected the plates of another condenser of 0.05 microfarad capacity. And so on, as marked on the instrument. Connecting any number of condensers in parallel is equivalent to adding the areas of their plates. That is, the capacity of a number of condensers in parallel equals the sum of the capacities of the separate condensers. With the plugs arranged as in the figure, the capacity between the binding posts is $0.05 + 0.05 + 0.2 + 0.2 + 0.5$ microfarads.

58. Self-Induction. — Whenever the current in a circuit is changed there is an electromotive force induced in the circuit which tends to oppose the change. This phenomenon is called self-induction. The magnitude of the electromotive force of self-induction depends upon the shape of the circuit but is always

proportional to the rate of change of the current. Thus, the electromotive force of self-induction

$$E_s = -L\frac{\Delta i}{\Delta t},$$

where the constant L is called the self-inductance or the coefficient of self-induction, and the negative sign is inserted to indicate that the direction of the induced electromotive force is opposite to that of the change of current.

The total electromotive force in a circuit is equal to the sum of the IR drops plus the electromotive force of self-induction. Thus, if the current is rising in value at the rate $\Delta i/\Delta t$,

$$E = IR + L\frac{\Delta i}{\Delta t}. \tag{195}$$

If falling at the same rate,

$$E = IR - L\frac{\Delta i}{\Delta t}. \tag{196}$$

And if the current is steady,

$$E = IR. \tag{197}$$

59. Standard of Self-Inductance. — A variable standard of self-inductance is shown in Fig. 93. The circuit consists of two circular coils in series — a fixed coil B within which is the coil A capable of rotation about the vertical diameter. When the two coils are coplanar and the current flows through both in the same direction, the interlinking of the two by the magnetic field is maximum and the inductance is maximum. As the movable coil is rotated from this position, the magnetic interlinkage is diminished until, when the coil is 180° from the former position, the magnetic fields due to the two coils are in opposite directions and the inductance is minimum. A pointer attached to the movable coil passes over a divided scale C graduated to indicate directly the coefficient of self-induction of the circuit.

60. Mutual Induction. — A change in the magnitude of the current in a circuit causes the development of an electromotive force in any neighboring circuit. This phenomenon is called

mutual induction. The electromotive force of mutual induction is directly proportional to the rate of change of the current in the primary circuit. Thus,

$$E_m = -M \frac{\Delta I}{\Delta t}, \tag{198}$$

where the constant M, called the mutual inductance or the coefficient of mutual induction, depends upon the shape, size, and num-

FIG. 93.

ber of turns of wire in each circuit, and the medium surrounding them. A pair of circuits is said to have unit mutual inductance when unit electromotive force is induced in one by a unit change of current per second in the other.

Now the charge which in time Δt passes any point of a conductor traversed by a current I is

$$Q \,[= I \Delta t] = \frac{E}{R} \Delta t.$$

So that the charge set into motion in the secondary circuit of resistance R due to a change of current ΔI in the primary circuit is (198)

$$Q = -M\frac{\Delta I}{R}. \tag{199}$$

61. Standard of Mutual Inductance.—It is possible to compute the mutual inductance of a pair of coils of certain shapes and arrangements with respect to one another. The arrangement for which the computation is most simple is a primary consisting of a long close spiral of many turns of small diameter, with a coaxial secondary of few turns occupying a short space at the middle of the long primary. The secondary coil may be either outside or within the primary coil. Such a standard of mutual inductance is illustrated in Fig. 94. It comprises a secondary coil of known number of turns and diameter overwrapped by a long primary coil consisting of a single layer of turns of known number and diameter.

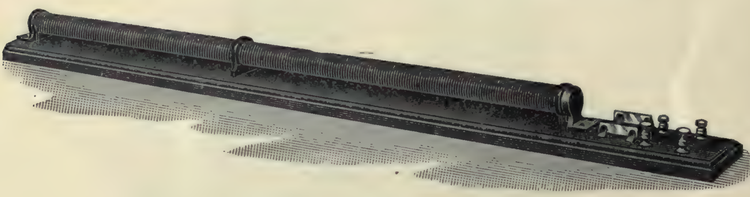

FIG. 94.

We will now deduce an expression for the value of the mutual inductance of such a pair of coaxial coils. Now whenever a circuit is subject to a change of flux-turns ΔN in time Δt, there is induced an electromotive force

$$E = -\frac{\Delta N}{\Delta t}.$$

If this change of flux-turns be due to a change of current ΔI in a neighboring circuit, we have (198)

$$-\frac{\Delta N}{\Delta t} = -M\frac{\Delta I}{\Delta t},$$

or,

$$M = \frac{\Delta N}{\Delta I}. \tag{200}$$

Let n_1 be the total number of turns in the primary coil, l_1 the length of the coil, and n_1' $[= n_1/l_1]$ the number of turns in the primary per unit length of the coil. Let n_2 be the total number of turns in the secondary coil, r_2 the radius of the secondary coil, and A the area of the secondary coil.

If the current in the primary coil changes Δi absolute units, there will be a change of the magnetic field strength at the center of the coil (193)

$$\Delta H = 4\,\pi n_1'\Delta i.$$

Hence in the secondary coil there is a change of magnetic flux

$$\Delta\Phi\,[= A\Delta H] = A\,4\,\pi n_1'\Delta i_1 = 4\,\pi^2 r_2^2 n_1'\Delta i_1,$$

and a change of the flux-turns

$$\Delta N\,[= n_2\Delta\Phi] = 4\,\pi^2 r_2^2 n_1' n_2 \Delta i_1.$$

Consequently (200),

$$\begin{aligned} M\left[= \frac{\Delta N}{\Delta i_1}\right] &= 4\,\pi^2 r_2^2 n_1' n_2 \qquad \text{C. G. S. units} \\ &= 0.4\,\pi^2 r_2^2 n_1' n_2 \qquad \text{henrys.} \end{aligned} \tag{201}$$

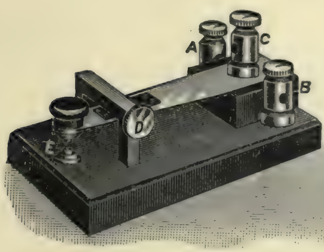

FIG. 95.

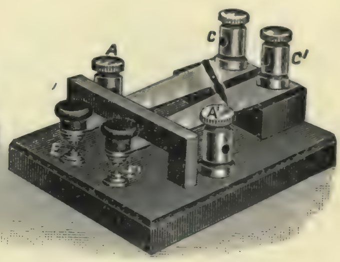

FIG. 96.

62. Keys and Switches. — For the convenient making, breaking, and reversing of currents, and the charging and discharging of condensers, various keys and switches are in common use. Common forms of tap keys for the simple making and breaking of small currents in one or two circuits are shown in Figs. 95 and 96. In the latter figure two tap keys are mounted on the same

base. The binding post A is connected under the base to the contact point E, and A' to E'.

For reversing a current, the reversing switch, Figs. 97 and 98, is usually employed. This consists of two copper knife blades, insulated from one another, and capable of being rocked to the right or left. By rocking the pair of blades, either pair of ends can be brought into contact with a corresponding pair of jaws attached to binding posts. These binding posts are connected by wires beneath the base as shown in Fig. 98. With the blades to the left as in Fig. 97, current from the main line, Fig. 98, would follow the path $abGb'a'$; whereas if the blades be thrown to the right, the current will follow the path $ac'b'Gbca'$.

FIG. 97.

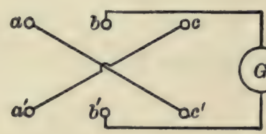

FIG. 98.

By removing the wires under the base that connect the diagonally opposite binding posts, this reversing switch becomes a double-pole double-throw switch. When high insulation is required, all six binding posts are put on petticoated vulcanite pedestals as in Fig. 99. This figure shows how, by making one blade a little deeper than the other, when the handle is thrown to the right, connection can be made to one blade a little earlier than to the other.

Another charge-and-discharge key that can also be used as a single-pole double-throw switch when high insulation is required, but not instantaneous make-and-break, is shown in Fig. 100.

When a high insulation key is required that permits a practically

instantaneous charge and discharge, the Kempe charge-and-discharge key, Fig. 101, is available. This consists of a lever, one end of which plays between two platinum-faced stops attached to the

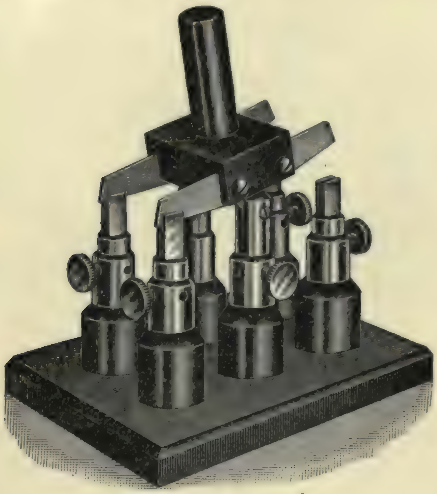

FIG. 99.

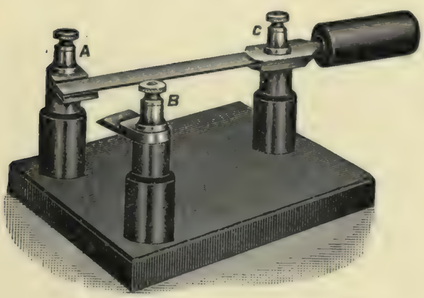

FIG. 100.

binding posts A and B. When the other end is caught under the hook of the trigger marked "insulate," the lever connects B and C. On pressing this trigger, the lever is at first released and afterward caught by the hook on the trigger marked "discharge."

In this position the connection between B and C is broken. On now pressing the trigger marked "discharge," the lever flies into the position shown in the figure in which A and C are connected.

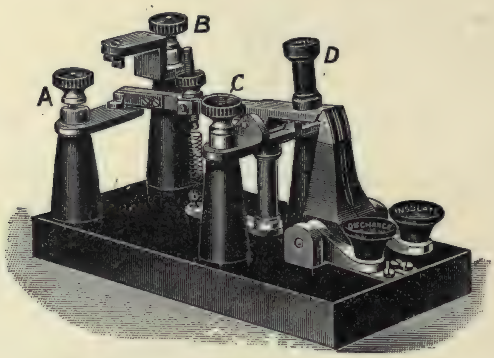

FIG. 101.

A pin reaching from the "discharge" trigger to the "insulate" trigger is so arranged that when the lever connects B and C, this connection can be broken and the connection between A and C instantly made by simply pressing the "discharge" trigger.

Exp. 85. Study of a Shunted Galvanometer

THEORY OF EXPERIMENT. — Read Arts. 33, 37, and 38.

MANIPULATION. — Connect the galvanometer G, shunt S, external resistance R, galvanic cell B, and reversing key K, as shown in Fig. 102. With the shunt resistance infinite, adjust R till the galvanometer deflection is about 15 cm. Reverse the current by means of the key and note the deflection. The mean of these two deflections is to be taken as the galvanometer deflection without shunt. Without altering the external resistance, find the deflections for a series of shunt resistances extending from infinity to zero.

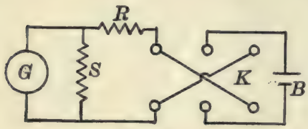

FIG. 102.

With deflections as ordinates and shunt resistances as abscissæ, plot a curve coördinating galvanometer deflections and shunt resistances. From the point on the deflection axis corresponding to infinite shunt resistance, draw a line parallel to the shunt resistance axis. This will be asymptotic to the curve.

With a shunt resistance equal to the galvanometer resistance, the galvanometer current, and hence the galvanometer deflection, would be half the value it would be if the shunt resistance were infinite. Consequently, on the above curve, the resistance corresponding to a deflection of one-half that given with a shunt of infinity equals the resistance of the galvanometer.

From the curve, find the resistance of the galvanometer. Also find the resistance of the shunt that would increase the range of the galvanometer 10 times; 1000 times.

Exp. 86. Determination of the Sensitivity of a Galvanometer

Theory of the Experiment. — Read Arts. 33, 38, and 43. From definition, the figure of merit

$$F = \frac{i_g}{d},$$

where i_g represents the current in the galvanometer expressed in amperes, and d is the deflection in millimeters when the scale is distant one meter.

If a galvanic cell of known electromotive force be available, together with two resistance boxes, the value of i_g that gives an observed deflection d can be measured and thus the value of F determined. In Fig. 103, representing the external resistance, the shunt resistance, the galvanometer resistance, and the battery resistance by R, s, g, and b, respectively, the electromotive force by E, and the total current by I, we have

$$I = \frac{E}{R + b + \dfrac{sg}{s+g}}$$

and the current through the galvanometer

$$i_g\left[= I\frac{s}{s+g}\right] = \frac{E}{\left(R + b + \dfrac{sg}{s+g}\right)}\frac{s}{(s+g)}.$$

Whence,

$$F\left[= \frac{i_g}{d}\right] = \frac{E}{\left(R + b + \dfrac{sg}{s+g}\right)}\frac{s}{(s+g)\,d}. \tag{202}$$

During an experiment, the resistances R and s would be observed at the same time as the deflection d. If the value of b is small compared with g and R, it may be neglected without appreciable error. The values of E and g must be determined.

The galvanometer resistance g may be determined by the Half Deflection Method now to be described. In Fig. 103, a resistance box R_1 is in series with a galvanic cell B. By joining the terminals a and b of the galvanometer to different lugs on the resistance box, various deflections can be obtained. Let the position of a and b be adjusted till the galvanometer deflects nearly across the scale. Then introduce into the galvanometer circuit such a resistance R_2 that the deflection will be one-half its previous value. Representing the potential difference between a and b in the first case by V and the potential difference in the second case by V', the current in the battery by I, and the current in the galvanometer in the first case by I_g, and the current in the galvanometer in the second case by I_g', we have (155)

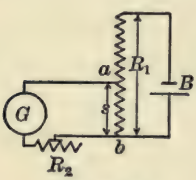

FIG. 103.

$$I_g\left[= \frac{V}{g}\right] = \frac{I\dfrac{gs}{g+s}}{g} \tag{203}$$

and

$$I_g'\left[= \frac{V'}{g+R_2}\right] = \frac{I\dfrac{(g+R_2)}{g+s+R_2}}{g+R_2} \tag{204}$$

Dividing each member of (203) by the corresponding member of (204),

$$\frac{I_g}{I_g'}[=2] = \frac{g + s + R_2}{g + s}.$$

Whence,

$$g = R_2 - s. \tag{205}$$

MANIPULATION. — Find the resistance of the galvanometer by the Half Deflection Method. With a fairly sensitive galvanometer and a single dry cell, the resistance R_1, Fig. 103, will be from 5000 to 10,000 ohms and the resistance between a and b will be from one to two ohms.

Measure the electromotive force of the cell by joining directly to a high-resistance voltmeter.

Now connect the apparatus as shown in Fig. 103. Adjust R and s so as to get a large deflection. Reverse the current through the galvanometer by means of the switch K and note the deflection. The mean of these two deflections is the value of d to be used in (202).

Note the resistance R and s. The battery resistance b is so small compared with R that it can be neglected in (202) without sensibly affecting the value of F.

In the same manner obtain values of F for a series of not less than five different values of d. Plot a curve coördinating deflections and figure of merit.

From the average value of F compute by (169) and (170) the megohm and voltage sensitivities.

Exp. 87. Determination of the Resistance of a Wire by Means of a Slide-wire Bridge

THEORY OF THE EXPERIMENT. — Read Arts. 44, 46–48.

MANIPULATION. — Put the unknown resistance in the gap marked R_1, Fig. 104, and a resistance box in the gap R_3. Connect the terminals of a galvanometer to B and D. Close the gaps shown on either side of B with short pieces of wire.

With a certain resistance R_3, move the sliding contact D back and forth till a point is found where there is no galvanometer deflection on pressing the battery and galvanometer keys. If this point be not near the middle of the scale, alter R_3 till the point of balance is near the middle of the scale. Note the position of

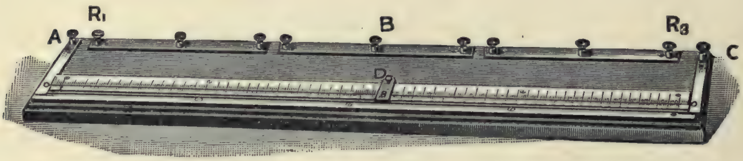

FIG. 104.

D and the resistance R_3, and by means of (174) compute the approximate resistance R_1.

Interchange the positions of the resistance box and the specimen and again find the approximate value R_1.

If these two values are not practically equal, the bridge has end or tapping errors and the resistance of the specimen must be determined by substituting the already observed data in (178).

Make two determinations for each sample furnished.

Exp. 88. A Study of the Laws of Series and Parallel Resistances by Means of a Box Bridge

THEORY OF THE EXPERIMENT. — Read Arts. 37 and 46.

MANIPULATION. — Connect the galvanic cell, galvanometer, and unknown resistance to the box bridge, using short thick wire to connect the unknown resistance. First, with the ratio arms equal, adjust the rheostat arm until the galvanometer gives the minimum deflection on closing the battery key and then the galvanometer key. This adjustment is complete when, on changing the resistance of the rheostat arm by one ohm, in one direction, the galvanometer deflection is increased; while on changing the resistance by one ohm, in the other direction, the galvanometer deflection is

reversed. From the values of the resistance of the rheostat and ratio arms, the approximate resistance of the specimen can be determined by (173). Knowing the approximate value of the resistance, determine the setting of the ratio arms that will make the rheostat arm read to four digits and find the more accurate value of the resistance.

After making a careful determination of the resistance of each of three wires, measure the resistance of the three in series and also the resistance of the three in parallel.

Using the known resistances of the separate wires, compute the resistance of the three in series and also the resistance of the three in multiple. Compare these computed values with the experimentally determined values.

Exp. 89. Calibration of a Resistance Thermometer

Theory of Experiment. — Read Art. 46. The mercury-in-glass thermometer is unavailable for the measurement of temperatures much below $-30°$ C. or above $+300°$ C. Although the gas thermometer can be used for any temperature for which a suitable material to construct the bulb can be found, it is such a large awkward instrument, and the difficulties of the manipulation are so considerable, that it is suitable only for standardizing more convenient types of thermometer. Since the electrical resistance of metals varies continuously with the temperature according to definite laws, and since the accurate measurement of resistance is attended with no considerable difficulty, thermometers depending upon this change of resistance are in common use for measuring high and low temperatures.

It has been shown by experiment that if R_0 represent the resistance of a piece of pure platinum, nickel, or of certain other metals, when at 0° C., then the resistance at $t°$ C. is expressible by the equation

$$R_t = R_0 (1 + at + bt^2), \qquad (206)$$

where a and b are constants for the particular material.

If the resistances of the wire at any temperatures $t_1°$ C., $t_2°$ C., and $t_3°$ C. be R_1, R_2, and R_3, respectively, then we may write

$$R_1 = R_0\,(1 + at_1 + bt_1^2),$$
$$R_2 = R_0\,(1 + at_2 + bt_2^2),$$
$$R_3 = R_0\,(1 + at_3 + bt_3^2).$$

After measuring the resistances R_1, R_2, and R_3, at the known temperatures t_1, t_2, and t_3, respectively, the constants R_0, a, and b can be determined by means of the three equations above. These constants substituted in (206) give an equation that coördinates the resistance and temperature of the particular wire employed. That is, we have the equation of the calibration curve of the resistance thermometer. This equation can be used for the computation of values of R corresponding to any assumed temperatures. From a series of values of resistances and corresponding temperatures the required calibration curve can be plotted.

Manipulation. — The resistance thermometer consists of a coil of fine wire, usually nickel or platinum, enclosed in a suitable bulb, with leads joined to binding posts on the outside of the bulb. To eliminate any error due to uncertainty regarding the temperature of the leads, provision must be made to obtain the resistance of the coil alone.

Fig. 105.

For example, the coil may be arranged as in Fig. 105, with a "dummy lead" mB joined to one end of the coil. The three leads, Cp, nB, and mB, are of the same length, material, and resistance. The resistance of the coil alone can be obtained by subtracting the resistance between n and m from the resistance between C and n.

Fill a small can with pieces of ice no larger than a pea and cover the ice with water. Immerse the bulb of the resistance thermometer in this ice bath. By means of either the slide wire bridge or the box bridge measure the resistance between C and n and also that between m and n. The difference between these two resistances is the resistance of the coil at 0° C.

Proceeding in the same manner find the resistance of the coil

when in the steam from water boiling under atmospheric pressure, and also when in the vapor from naphthalene boiling under the same pressure. Under atmospheric pressure the vapor from boiling naphthalene is 219° C. In making these resistance measurements one must press the keys for the shortest possible time to avoid any heating of the coil due to the current.

FIG. 106.

To obtain the boiling point of water a vessel such as is illustrated in Fig. 106 is very satisfactory. By means of the water manometer M, any difference of pressure between the steam inside and the air outside can be observed. If the barometric pressure be H millimeters of mercury and the manometer indicates a pressure of d millimeters of water (or, $d \div 13.6$ millimeters of mercury), then the total pressure on the surface of the boiling water is

$$H + (d \div 13.6) \text{ mm. of mercury.}$$

FIG. 107.

The temperature of water boiling under various pressures is given in Table 3.

To obtain the boiling point of naphthalene the apparatus illustrated in Fig. 107 is convenient. The naphthalene contained in an aluminium tube is heated by a current-carrying conductor. To prevent drops of liquid naphthalene that condense on the upper part of the thermometer tube from running down, over the bulb, as well as to diminish the loss of heat by radiation, the thermometer bulb is enclosed in a thin aluminium shield.

If the bulb be of quartz or porcelain it will require considerable time for the coil to attain the temperature of the ice, steam, or naphthalene vapor. After the bulb has been immersed, take resistance readings till they remain constant for five minutes. These constant values are the ones to be used in the computation.

From the values of the three resistances determined at the three known temperatures, compute the constants in (206). Substitute these constants in (206), and by means of the empirical equation thereby obtained, compute the resistances of the coil at various temperatures at 100° intervals from $-50°$ C. to 300° C. With these values of temperatures as abscissæ, and resistances as ordinates, plot the calibration curve of the given resistance thermometer.

Exp. 90. Determination of the Conductivity of a Salt Solution at Different Temperatures

Theory of the Experiment. — Read Arts. 46, 47, 49, and 50. Electrolytic resistance measurements are made by means of the Wheatstone bridge method, using an alternating current and a telephone receiver. When determinations of only moderate precision are required a small induction coil can be used to supply the

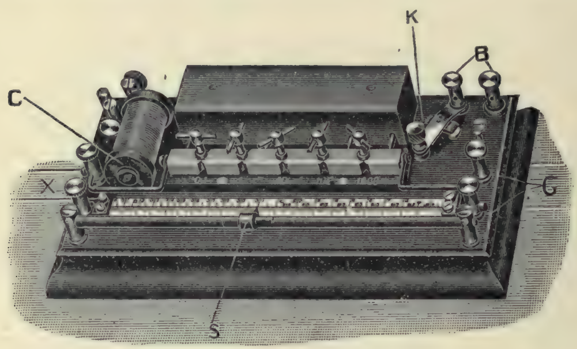

Fig. 108.

alternating current. A very compact slide wire bridge is shown in Fig. 108. The induction coil C is operated by a small galvanic cell connected to the terminals B. A telephone receiver is joined to the terminals G, and the resistance to be measured is joined to the terminals X. Known resistances are introduced into one arm of the bridge by removing the plugs shown back of the slide wire.

For determinations requiring greater precision an alternating current that has a sine-wave form must be used. For this purpose small high-frequency alternating-current dynamos are available. By substituting such an alternator for the induction coil there will be less electrolytic polarization and a greater precision in the determination.

For work of the highest degree of precision the impressed current must have a true sine-wave form and the capacity and inductance of the bridge system must be balanced. In the present determination, this degree of precision will not be attempted.

In determining conductivities of electrolytes it is common to take a one-fiftieth molecular normal * solution of potassium chloride as a standard for comparison. At 18° C., the conductivity of this solution is 0.002397 reciprocal ohms per centimeter cube. The change in conductivity per degree centigrade change of temperature is $52(10^{-6})$ reciprocal ohms per centimeter cube.

MANIPULATION. — Fill the carefully washed and dried cell with the standard solution and place in a water bath. When a mercury-in-glass thermometer in the solution indicates a constant temperature, adjust the slider of the slide-wire bridge for minimum sound. Except when a current having a pure sine-wave form is used, and the inductance and capacity of the circuit are accurately balanced, the slider can be moved some distance without appreciably altering the loudness of the sound heard in the telephone. In this case the mid-position is to be taken as the setting. Use the average of not less than five such settings to compute the resistance. Record the temperature of the solution.

Empty the cell, carefully wash and dry it, and fill with the sample under test. Measure the resistance of the sample at not less than five different temperatures differing from one another by from three to five degrees.

* A solution of the concentration that would be obtained by dissolving the number of grams of solute equal to its molecular weight in enough water to make one liter of solution is called the molecular normal solution of the given solvent. Many substances are not sufficiently soluble to form molecular normal solutions. A one-fiftieth molecular normal solution of KCl contains 0.02 (39 + 35.5) gm. of KCl and enough water to make one liter of solution.

By means of (184) calculate the conductivities of the solution at the various temperatures. Plot a curve coördinating conductivity and temperature.

Exp. 91. The Determination of a Low Resistance by the Direct Deflection Method

THEORY OF THE EXPERIMENT. — Read Art. 49. For resistances less than one-tenth of an ohm the Wheatstone bridge method is inaccurate on account of the resistance of the wires connecting the specimen to the bridge. A fall of potential scheme obviates this difficulty. This consists in comparing the fall of potential along the specimen under test with the fall of potential along a known resistance of about the same magnitude, the two resistances being traversed by the same current.

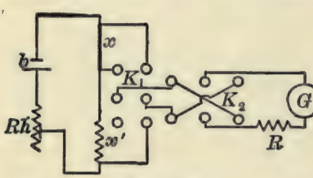

FIG. 109.

In Fig. 109 the unknown resistance x, the standard resistance x', storage cell b, and rheostat Rh are joined in series. By means of a double-throw double-pole switch K_1 the potential difference at the terminals of x or of x' may be impressed on the galvanometer G. The current through the galvanometer may be reversed in direction by means of the reversing key K_2. The resistance of the galvanometer together with an added resistance R must be so great that the current in the main circuit shall not be appreciably altered by connecting the galvanometer to the ends of either x or x'.

With the same current in x and x', the potential differences at their terminals will be directly proportional to their resistance. That is,

$$\frac{V_x}{V_{x'}}\left[=\frac{xi_x}{x'i_{x'}}\right]=\frac{x}{x'},$$

where V_x and $V_{x'}$ represent the potential differences at the ends of the resistances x and x', respectively.

Since the deflection produced by connecting a galvanometer to the terminals of resistances of nearly the same value and traversed

by the same current will be nearly equal, in this experiment almost any high-resistance galvanometer will give deflections proportional to the potential differences at the terminals of x and x'. Thus,

$$\frac{d_x}{d_{x'}}\left[=\frac{V_x}{V_{x'}}\right]=\frac{x}{x'}, \tag{207}$$

where d_x and $d_{x'}$ are the deflections produced by placing the galvanometer across x and x', respectively.

In this experiment will be determined the resistivity of a piece of wire and also the contact resistance of a knife switch.

MANIPULATION. — Connect the apparatus as shown in Fig. 109. To hold a sample of wire, the clamp shown in Fig. 110 is convenient. The sample is held in the binding posts BB' and current

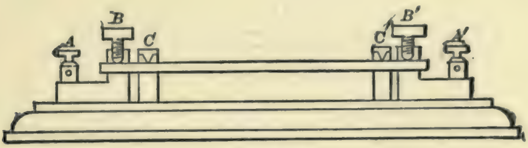

FIG. 110.

is led into the sample through the binding posts AA'. The galvanometer is connected to the sample by means of the weighted knife-edges C, C'. The distance between these contacts should be measured and be about 50 cm. Measure the diameter of the wire in several places by means of a micrometer caliper and use the average in determining the area of cross section.

For a standard use a resistance x' of the same order of magnitude as the resistance x under test. If the galvanometer have a resistance of 500 ohms or more, no added resistance R, Fig. 109, need be employed. With a galvanometer of about 100 ohms a series resistance R of not less than 200 ohms should be used.

Adjust the rheostat Rh till the deflections d_x and $d_{x'}$ are large but well on the scale. Take several readings, both right and left, for both the known and the unknown resistance. Calculate the resistance by means of (207) and the resistivity by means of (180).

To determine the contact resistance of a knife switch, send the current through the switch and take the fall of potential across the contact from leads soldered to each side and as close to the contact as possible.

Exp. 92. Determination of Resistances by Means of the Ammeter-Voltmeter Method

Theory of the Experiment. — When great precision is not required, a resistance can be readily determined by means of an ammeter and a voltmeter.

If the resistance to be measured be small — say less than one ohm — the voltmeter V, ammeter A, unknown resistance R, and source of electromotive force are connected as shown in Fig. 111. In this case, we have from Ohm's law,

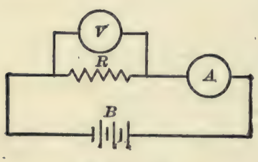

Fig. 111.

$$R = \frac{V_R}{I_R},$$

where V_R is the potential difference at the ends of the unknown resistance and I_R is the current through R. When the resistance R is small compared with that of the voltmeter, an inappreciable current will traverse the voltmeter and the current I_R through R will equal approximately the current I through the ammeter. That is,

$$R\left[=\frac{V_R}{I_R}\right] \doteq \frac{V_R}{I}. \qquad (208)$$

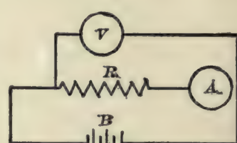

Fig. 112.

If the resistance to be determined be more than one ohm, the above approximation will probably not be sufficiently precise. By putting the voltmeter around both the unknown resistance and the ammeter, Fig. 112, we have

$$R + R_A = \frac{V_{RA}}{I}.$$

Since the resistance of an ammeter is only a few thousandths of an ohm, R_A may usually be neglected, and we may write

$$R \doteq \frac{V_{RA}}{I}. \tag{209}$$

Manipulation. — By the methods above described, determine a resistance that is known to be less than one ohm and also one known to be 50 or more ohms.

Exp. 93. Determination of a Resistance by the Voltmeter Method

Theory of the Experiment. — A voltmeter of known resistance in connection with a source of electromotive force of negligible resistance can be used to determine a resistance of the magnitude of the voltmeter resistance. Though not susceptible of great precision, the method is so easily applied that it is of considerable utility. For example, it is often used in central stations to measure insulation resistances. In this case, a switchboard voltmeter and direct-current dynamo are employed.

If the voltmeter be connected directly to the terminals of the source of electromotive force, the potential difference, V_1, at the voltmeter terminals will have the value

$$V_1 = i_1 r_v, \tag{210}$$

where i_1 is the current in the voltmeter and r_v is the resistance of the instrument.

If, now, the unknown resistance of value R be connected in series with the voltmeter, the potential difference, V_2, at the terminals of the instrument will be less than that before by the fall of potential through the resistance R. That is,

$$V_2 = V_1 - i_2 R,$$

or,

$$V_1 - V_2 = i_2 R. \tag{211}$$

Dividing each member of (210) by the corresponding member of (211),

$$\frac{V_1}{V_1 - V_2} = \frac{i_1 r_v}{i_2 R}.$$

But since the current in any conductor is proportional to the potential difference at the terminals,

$$\frac{i_1}{i_2} = \frac{V_1}{V_2},$$

so that the preceding equation becomes

$$\frac{V_1}{V_1 - V_2} = \frac{V_1 r_v}{V_2 R}.$$

Consequently,

$$R = \left(\frac{V_1 - V_2}{V_2}\right) r_v, \tag{212}$$

if the resistance of the source of electromotive force is negligible compared with R.

Manipulation. — Using a 150-volt voltmeter in connection with the current from a direct-current dynamo, find the value of a resistance that is of the order of 10,000 ohms.

Exp. 94. Determination of the Insulation Resistance of a Cable by the Substitution Method

Theory of the Experiment. — Read Art. 39. The substitution method is especially suited to the measurement of high resistances. If a sensitive galvanometer of resistance g be joined in series with a standard resistance of high value r_s and a battery or dynamo of electromotive force E and resistance b, there will be a current through the galvanometer of the value

$$I_1 = \frac{E}{r_s + b + g}.$$

If an unknown resistance r_x be substituted for the standard, the current will have the value

$$I_2 = \frac{E}{r_x + b + g}.$$

And if the galvanometer deflections are proportional to the galvanometer currents, we will have

$$\frac{d_1}{d_2}\left[= \frac{I_1}{I_2}\right] = \frac{r_x + b + g}{r_s + b + g}.$$

Now, the magnitude of b is so small compared with r_x and r_s that it may be neglected. And if r_x and r_s are sufficiently large compared with g, the latter can also be neglected. Whence, in this case,

$$r_x \doteq r_s \frac{d_1}{d_2}. \tag{213}$$

Manipulation. — For a lead-covered cable of five meters length use a standard resistance r_s of about 100,000 ohms, a galvanometer of high sensitivity, an Ayrton shunt, and a 110-volt direct-current dynamo.

Prepare the ends of the cable by peeling off the sheathing and the insulation so that the copper conductor protrudes beyond the insulation about one inch, and the insulation extends beyond the sheathing about the same distance.

Insert the insulation resistance in the place marked r, Fig. 64, by connecting the sheathing to one lead wire and the copper conductor to the other.

With the Ayrton shunt set for smallest galvanometer deflections, press the key and observe the deflection. If the deflection be less than one-tenth the scale, change the shunt resistance till a suitable deflection is produced. Note this deflection.

Again set the Ayrton shunt for minimum deflection and substitute the standard resistance for the cable. On pressing the key, if a suitable deflection is not obtained, change the resistance of the Ayrton shunt. Note the deflection.

By means of (160) reduce both deflections to the values they would have had if the cable and the standard had been connected from A to C, Fig. 64, across the entire shunt. These are the values to be used in (213).

Exp. 95. Study of a Galvanic Cell

Theory of the Experiment. — The electromotive force of a cell depends only upon the actions within it. When the actions within the cell are altered by polarization, the electromotive force is changed. The potential difference at the terminals of any generator depends upon the electromotive force and internal

resistance, and upon the resistance of the external part of the circuit.

Representing the electromotive force, the terminal potential difference, and the internal resistance by E, V, and b, respectively, and the external resistance of the circuit by R, we have, by Ohm's law,

$$I = \frac{E}{R + b} = \frac{V}{R}. \tag{214}$$

Whence,

$$V = \frac{E}{\frac{b}{R} + 1}. \tag{215}$$

From this equation it is seen that as the ratio of the internal resistance of a generator to the resistance of the external part of the circuit approaches zero, the potential difference at the terminals approaches in value the electromotive force. When the external resistance is infinite, that is, when a generator is on open circuit, the terminal potential difference equals the electromotive force.

From (215) we also have

$$b = R\left(\frac{E}{V} - 1\right). \tag{216}$$

In this experiment a dry cell is to be discharged and allowed to recover several times, and the relations plotted between time and terminal potential difference, time and electromotive force, time and current, time and internal resistance.

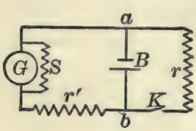

Fig. 113.

Manipulation. — First, measure the electromotive force of the cell under test with a millivoltmeter. Then connect the cell under test B, with shunt S, resistance r' of about 20,000 ohms, resistance r of about 5 ohms, and key K, as in Fig. 113. Adjust the galvanometer shunt for large deflections that remain on the scale.

Since the galvanometer deflections are proportional to the

potential differences at the battery terminals, we may write (216) in the form

$$b = R\left(\frac{d}{d'} - 1\right),$$

where d and d' represent the deflections produced when the key K is open and when it is closed, respectively, and R represents the total external resistance of the circuit. If the resultant resistance from a to G to b be represented by R', we will have

$$R = \frac{rR'}{r + R'}.$$

But in the present experiment in which R' is greater than 20,000 ohms, and r is only about 5 ohms, there will be no appreciable error in assuming that $R = r$. We then have

$$b \doteq r\left(\frac{d}{d'} - 1\right). \tag{217}$$

With the key open, read the deflection d; close the key and read d'. At the end of two minutes again read d', then open the key long enough to again read d. Continue at two-minute intervals until d' has fallen to about one-half of its original value. Now, with the key open, take values of d at two-minute intervals until the value becomes constant.

Reduce the galvanometer deflections to volts by means of the voltmeter reading taken at the beginning of the experiment and the first galvanometer deflection taken with the key open.

By means of (217) find the battery resistance corresponding to each observation of d'. By means of Ohm's law find the current through the cell corresponding to each observation of d'.

Plot on one sheet curves coördinating the following quantities: (*a*) time and electromotive force on discharge; (*b*) time and terminal potential difference on discharge; (*c*) time and electromotive force on recovery; (*d*) time and original electromotive force. On a second sheet plot curves coördinating: (*e*) time and battery resistance; (*f*) time and current.

Discuss the curves.

Exp. 96. A Study of the Potential Galvanometer

Theory of the Experiment. — Read Arts. 41 and 42. In this experiment a double-range voltmeter is to be studied. First, the resistance of the instrument for the lower range is to be determined by the Half Deflection Method. Using this value, the resistance of the multiplier required to increase the range of the instrument by a factor of 10 is to be computed. A multiplier of this resistance is to be placed in series with the low-range posts of the instrument and a source of sufficient electromotive force to give a deflection nearly across the scale, and the deflection noted. The result is to be checked by comparison with the deflection produced when the high-range posts of the instrument are connected directly to the same source.

Manipulation. — For a voltmeter of ranges 0–3 volts and 0–30 volts, use a dry cell and resistance box to find the resistance between the low-range posts. To produce a nearly full-scale deflection when either the computed multiplier or the multiplier within the instrument is in circuit, use a storage battery of about 40 volts and a rheostat.

Problem. — The poles of a storage battery of electromotive force ten volts and resistance one ohm are joined by a uniform conductor 1900 ft. long of resistance 19 ohms. Compute the potential difference between two points on the conductor 1000 ft. apart. Compute the potential difference between the same two points when joined by an instrument of resistance ten ohms and also when joined by one of 1000 ohms.

Exp. 97. Calibration of a Ballistic Galvanometer

Theory of the Experiment. — Read Arts. 35, 55, 60, and 62. If when a quantity Q is discharged through a ballistic galvanometer there is produced a throw d, the ballistic constant has the value (152)

$$G = \frac{Q}{d}.$$

In Art. 35 it has been shown that the value of the ballistic constant depends upon the resistance of the galvanometer circuit. The object of this experiment is to determine the ballistic constant of

a galvanometer when the circuit has a resistance of infinity and also when some finite value R.

If the quantity of electricity sent through the galvanometer is supplied by the discharge of a condenser of capacity C charged to potential V, then, $Q = CV$, and the ballistic constant for the galvanometer on open circuit has the value

$$G_\infty = \frac{CV}{d}. \tag{218}$$

When the galvanometer is part of a closed circuit of total resistance r, a definite quantity can be discharged through it by changing the current in a neighboring circuit. If the pair of circuits have a mutual inductance M, then when the current in the primary circuit is changed by the amount Δi, the secondary circuit containing the galvanometer will be traversed by a charge given by (199). If this quantity produces a throw d, we have for the value of the ballistic constant of a galvanometer in a circuit of total resistance r,

$$G_r = \frac{M\Delta i}{rd}. \tag{219}$$

Manipulation. — With a millivoltmeter, measure the terminal potential difference V of a dry cell. Connect the cell, B, the ballistic galvanometer under test, a condenser of known capacity C, and a charge-and-discharge key as shown in Fig. 114. By means of the charge-and-discharge key, charge the condenser and then discharge it through the galvanometer and note the throw d. We then have from (218),

$$G_\infty = \frac{CV}{d}.$$

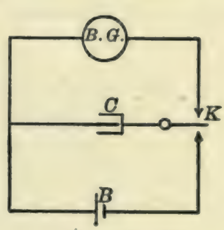

Fig. 114.

The ballistic constant, when the galvanometer circuit is of resistance R, will now be determined. Connect a battery of about 20 volts, ammeter A, ballistic galvanometer, resistance box R', rheostat R, standard of mutual inductance PS, and tap keys K_1,

and K_2 as shown in Fig. 115. The required known mutual inductance may be supplied by a primary standard as described in Art. 61, or by any pair of coils wound on a marble or wood spool the coefficient of mutual induction of which has been determined by an experimental comparison with a primary standard.

Set the resistance box R' at zero resistance. With K_2 closed, adjust the rheostat R so that when K_1 is opened or closed the galvanometer will give a throw about equal to that obtained in the previous part of the experiment. Record the value of this throw, the current, and the resistance of the galvanometer circuit ($r = R' + g$).

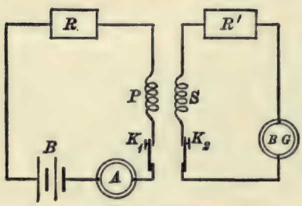

Fig. 115.

Repeat, with the resistance of the galvanometer circuit successively 2, 3, 4, 5, 8, and 10 times the first value. Throughout all of this work the throw d should be kept within 10 per cent of its first value by adjusting the rheostat R. Each galvanometer throw should be observed not less than five times.

With the known value of M, together with the observed values of Δi, r, and d, compute by means of (219) the ballistic constant of the galvanometer corresponding to each value of r.

Plot a curve coördinating G_r and r. A line parallel to the resistance axis through the point representing G_∞ will be asymptotic to this curve. Why?

Exp. 98. Verification of the Electromotive Force of a Standard Cell by Means of a Silver Voltameter and Potentiometer

Theory of the Experiment. — Read Arts. 32, 51–53. Suppose the cell under test to be put in the place of the standard cell of a potentiometer, and, assuming the value of the electromotive force of the cell, suppose the potentiometer to be adjusted to indicate potential differences directly. On substituting for the cell under test a source of known potential difference, and bringing the potentiometer to balance, the scale reading will correspond to the known potential difference unless the value of the electro-

motive force of the cell under test is different than the value assumed. If the true and the assumed values of the cell under test are E and E', respectively, and the true and the observed values of the potential differences of the second source are V and V', respectively, then,

$$\frac{E}{E'} = \frac{V}{V'},$$

or,

$$E = E' \frac{V}{V'}. \tag{220}$$

Manipulation. — Prepare the silver voltameter as described in Art. 32. Connect in series a storage battery B, silver voltameter SV, rheostat Rh, standard one-ohm resistance R, and ammeter A, as shown in Fig. 116. Adjust the rheostat till the ammeter indicates about one ampere.

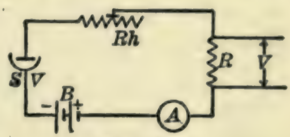

Fig. 116.

From the terminals of the standard resistance carry leads to the potentiometer, not shown in the figure. With the cell under test in the place assigned to the standard cell of the potentiometer, and assuming the value of the electromotive force of the cell, adjust the current in the potentiometer circuit so that the scale of the instrument indicates potentials directly. Substitute for the cell under test the leads from the terminals of R, Fig. 116, and rebalance the potentiometer.

After the apparatus has been thus adjusted, empty, wash, dry, and weigh the cathode of the silver voltameter. Refill and replace the voltameter and note the hour, minute, and second the circuit is closed. Measure the potential difference V' at the terminals of the standard resistance at five-minute intervals for about 45 minutes. Note the hour, minute, and second the circuit is opened. Empty, rinse, dry, and weigh the cathode.

From the mass and time of deposit compute the mean current by means of (150). From the current and resistance R compute by means of Ohm's law the average potential difference V impressed on the potentiometer. Representing the mean potentiometer

reading by V' and the assumed value of the electromotive force of the cell under test by E', compute the true electromotive force of the cell by (220).

The time occupied by one washing and drying of the cathode can be saved if the approximate resistance of the voltameter be known. When the time to be devoted to the experiment is brief, the instructor may furnish the student this value. In this case, the preliminary adjustment of the apparatus is made when a coil having the resistance of the voltameter occupies the place of the voltameter. After this adjustment, the voltameter, having a cathode of previously determined mass, is substituted for the coil of equal resistance and the experiment is continued as above described.

Exp. 99. Comparison of the Electromotive Force of a Cell with that of a Standard Cell by Means of a Potentiometer

Theory of the Experiment. — Read Arts. 51–53.

Manipulation. — For the working battery *W.B.*, Fig. 117, use a storage cell. For the slide wire use a uniform manganin wire

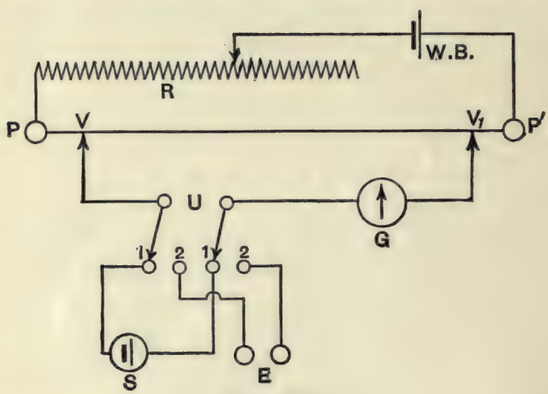

Fig. 117.

stretched over a scale 1.5 meters long divided into millimeters. One terminal of the galvanometer circuit is attached to the zero point V of the slide wire, and the other end is furnished with a

sliding contact V_1 like that used on the meter bridge. At U is inserted either a double-pole double-throw switch, Fig. 99, or a sliding pole changer as illustrated in Fig. 117. To one pair of posts of this switch connect the standard cell S with a resistance of not less than 100,000 ohms. And to the other posts connect the source E whose potential difference is required.

With about 100,000 ohms in series with the standard cell set the sliding contact at the scale division corresponding to the electromotive force of the standard cell. Adjust the rheostat in series with the working battery till the galvanometer deflection is zero. The sensitivity of the apparatus is greater when the resistance in series with the standard cell is less. Consequently, diminish this resistance first to about 10,000 ohms and readjust the rheostat. Then reduce this resistance to zero and readjust the rheostat. The potentiometer is now in adjustment for indicating potential difference directly. The rheostat must not be changed throughout the remainder of the experiment.

By throwing the switch U, substitute for the standard cell the one under test. With about 100,000 ohms in series with this cell, adjust the position of the slider till the galvanometer gives zero deflection. Reduce the resistance first to 10,000 ohms, rebalance, and then reduce the resistance to zero to obtain the final balance. Note the scale reading.

At the end of the experiment replace the standard cell and determine if the current through the slide wire has changed during the experiment.

Exp. 100. Calibration of a Voltmeter by the Compensation Method

THEORY OF THE EXPERIMENT. — Read Art. 51. In Fig. 118 a rheostat Rh is joined to the terminals of a battery of somewhat greater electromotive force than that which the voltmeter under test is designed to measure. The potential difference at the terminals of the voltmeter V can be varied from zero to the extreme limit of its range by moving the sliding contact V'. A galvanometer G, tapping key K, standard cell and high resistances R, r, and r' are

connected as shown. Since the points x and z are joined by wires of negligible resistance to the terminals of the voltmeter, the potential difference at the terminals of the voltmeter

$$V - V' = V_x - V_z. \tag{221}$$

Now when the current through any conductor is not varying, the potential difference between any two points of the conductor is directly proportional to the resistance between them. Whence,

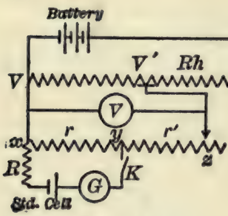

FIG. 118

$$\frac{V_x - V_z}{V_x - V_y} = \frac{r + r'}{r}. \tag{222}$$

From the diagram it is to be noted that the standard cell and the battery are in opposition. If the resistance r' be so adjusted that the potential difference $V_x - V_y$ due to the standard cell is exactly equal to that due to the battery, then the current through the standard cell will be zero, and

$$V_x - V_y = E,$$

where E is the electromotive force of the standard cell. Consequently, when this adjustment is made we have from (221) and (222),

$$V - V' [= V_x - V_z] = E\frac{r + r'}{r}. \tag{223}$$

MANIPULATION. — An inspection of the above equation shows that the computation of the potential difference at the terminals of the voltmeter under test can be readily simplified by making the number of ohms resistance in r one thousand times as great as the number of volts electromotive force E of the standard cell. By doing this

$$V - V' = 0.001\,(r + r'). \tag{224}$$

The number of ohms resistance r' should be about one thousand times the number of volts represented by the full scale of the voltmeter. The resistance R should be about 100,000 ohms in making preliminary settings.

Be careful that the + pole of the standard cell and the + pole of the battery are both directed to the right, or both to the left.

In making a setting, move V' to such a position on the rheostat that the voltmeter reading is about 0.1 of the full scale. Adjust r' till, on pressing the contact key, the galvanometer deflection is nearly zero. Reduce the resistance R to about 10,000 ohms and readjust r'. The resistance R may now be safely reduced to zero and the final adjustment of r' effected. Note the voltmeter reading and the resistances r and r'. Compute the true voltage by means of (224).

Move V' till the voltmeter indicates about 0.2 of the full-scale reading, make a balance, and take similar observations to the above.

In the same manner, make a series of settings at approximately equal voltage intervals throughout the range of the voltmeter.

Plot a curve coördinating voltmeter readings and corrections.

Exp. 101. Comparison of Potentials by Means of a Condenser and Ballistic Galvanometer

Theory of the Experiment.—Read Arts. 35 and 62. In this experiment the terminal potential difference of a dry cell is to be compared to that of a standard cell. If a condenser of capacity C be joined to the poles of the given cell of terminal potential difference V, the condenser will be charged with a quantity Q such that

$$Q = CV. \tag{225}$$

If this quantity be discharged through a ballistic galvanometer of ballistic constant G, there will be a throw d, such that (152)

$$Q = Gd. \tag{226}$$

If the same condenser be joined to the poles of a standard cell of terminal potential difference V_s, the condenser will be charged with a quantity Q_s, such that

$$Q_s = CV_s. \tag{227}$$

And if this quantity be discharged through the same ballistic galvanometer, there will be a throw d_s, such that

$$Q_s = Gd_s. \tag{228}$$

From (225) and (226),

$$CV = Gd. \tag{229}$$

and from (227) and (228),

$$CV_s = Gd_s. \tag{230}$$

Dividing each member of (229) by the corresponding member of (230), the required potential difference is seen to be

$$V = V_s \frac{d}{d_s}. \tag{231}$$

MANIPULATION. — Connect a mica condenser C of about two microfarads capacity, ballistic galvanometer BG, standard cell S, cell under test X, and the high-resistance switches K_1 and K_2 as shown in Fig. 119. A convenient form of double-throw switch suitable for K_1 is illustrated in Fig. 100. The charge-and-discharge key K_2 should be a Kempe key, Fig. 101.

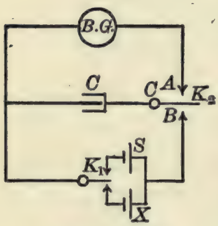

FIG. 119.

With the switch K_1 at the position to join one pole of the cell under test to one side of the condenser, press the handle of K_2 for a moment to charge the condenser and then quickly release the handle so as to discharge the condenser through the galvanometer. Note the galvanometer throw d_s.

In the same manner charge the condenser by connecting to the cell under test, discharge through the galvanometer, and note the throw d.

Knowing the terminal potential V_s of the standard cell, the terminal potential of the cell under test can be computed.

Exp. 102. Comparison of Capacities by the Direct Deflection Method

THEORY OF THE EXPERIMENT. — Read Arts. 35, 57, and 62. The object of this experiment is to compare the capacity of a given condenser with that of a standard condenser by means of the throws of a ballistic galvanometer produced by charging each

condenser to the same potential and then discharging it through the galvanometer.

In Fig. 120 are represented the condenser C_x under test, the standard condenser C_s, charging battery, ballistic galvanometer, and switches K_1 and K_2. With K_1 in contact with the standard and K_2 with the battery, the standard condenser will be charged with a quantity

$$Q_s = C_s V,$$

where V is the terminal potential difference of the battery. If now K_2 be raised into contact with A, this quantity will be discharged through the galvanometer, giving a throw d_s. If the ballistic constant of the galvanometer be G, we will have (152)

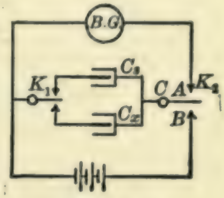

FIG. 120.

$$Q_s = Gd_s.$$

Whence,

$$C_s V = Gd_s. \tag{232}$$

If now this process be repeated for the condenser under test, we will have

$$C_x V = Gd. \tag{233}$$

Dividing each member of (233) by the corresponding member of (232), we obtain

$$C_x = C_s \frac{d}{d_s}. \tag{234}$$

MANIPULATION. — For the battery use dry cells — the number depending upon the sensitivity of the galvanometer and the capacity of the condensers. The standard condenser should be of about the same capacity as the condenser under test. A convenient switch for K_1 is the single-pole double-throw high-resistance switch shown in Fig. 100. For K_2, use the charge-and-discharge key shown in Fig. 101.

Exp. 103. Comparison of Capacities by the Method of Mixtures

THEORY OF THE EXPERIMENT.—Read Arts. 35, 57, and 62. Let the two condensers c_1 and c_2 be connected with two resistances R_1 and R_2, battery B, ballistic galvanometer G, and special double-pole double-throw switch KP as shown in Fig. 121. This special switch, Fig. 99, is so constructed that when the blades are thrown to the right, contact is made at M an instant before contact is made at P.

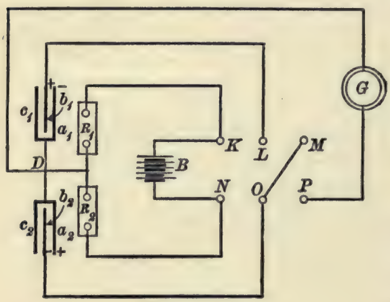

FIG. 121.

With the switch blades to the left, the condensers c_1 and c_2 are placed across the resistances R_1 and R_2, respectively. These resistances being in series are traversed by the same current, and therefore the IR drop in these coils will be proportional to the resistance. Thus, letting V_1 and V_2 represent the fall of potential across R_1 and R_2, respectively, we have

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}. \tag{235}$$

The two condensers of capacities C_1 and C_2 will become charged with quantities Q_1 and Q_2, respectively, such that

$$Q_1 = C_1V_1$$

and

$$Q_2 = C_2V_2.$$

Whence, from (235),

$$\frac{Q_1}{Q_2} = \frac{C_1R_1}{C_2R_2}. \tag{236}$$

On throwing the blades of the switch to the right, contact is first made at M, thereby causing the charges Q_1 and Q_2 to mix. If the charges were equal, the resultant charge will be zero. But if the charges were unequal, on making contact with P an instant later, the outstanding charge will traverse the ballistic galvanometer G.

If the resistances are adjusted until there is no galvanometer throw on charging the condensers, mixing, and discharging through the galvanometer, we know that $Q_1 = Q_2$. In this case (236) becomes

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}. \tag{237}$$

MANIPULATION. — For a charging battery use five dry cells. It is preferable to have the standard condenser of about the same capacity as the capacity being determined.

Exp. 104. Comparison of Self-Inductances

THEORY OF THE EXPERIMENT.—Read Arts. 36, 58, and 59. The object of this experiment is to determine the self-inductance of a given coil or circuit by comparison with that of a variable standard of self-inductance.

Consider a Wheatstone bridge circuit, two arms of which contain noninductive resistances R_2 and R_4, and two arms containing self-inductance and resistance as indicated in Fig. 122. Let first a constant electromotive force be impressed on the bridge and the resistances adjusted till a direct-current indicator in the bridge wire BD gives zero indication. When this adjustment is complete, we know from (173) that

$$R_1R_4 = R_2R_3.$$

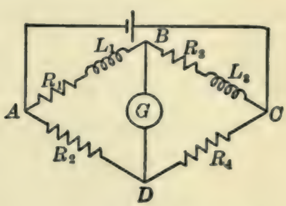

FIG. 122.

Let now a source of alternating electromotive force be substituted for the one previously used, and a detector of alternating current for the one previously used. The bridge will no longer be in balance. But the balance can be restored without altering any of the resistances by adjusting the inductance of the variable standard.

When the alternating electromotive force is used, we have (195)

$$V_A - V_B = i_1R_1 + L_1\frac{di_1}{dt}; \qquad V_B - V_C = i_3R_3 + L_3\frac{di_3}{dt};$$

$$V_A - V_D = i_2R_2; \qquad V_D - V_C = i_4R_4.$$

When in balance, $V_B = V_D$, so that

$$i_1R_1 + L_1\frac{di_1}{dt} = i_2R_2$$

and

$$i_3R_3 + L_3\frac{di_3}{dt} = i_4R_4.$$

Cross multiplying,

$$i_1R_1i_4R_4 + i_4R_4L_1\frac{di_1}{dt} = i_2R_2i_3R_3 + i_2R_2L_3\frac{di_3}{dt}.$$

Since the bridge is in balance, $i_1 = i_3$ and $i_2 = i_4$. And since the resistances are the same as they were when direct current was used, $R_1R_4 = R_2R_3$. Whence, the above equation reduces to

$$R_4L_1 = R_2L_3, \tag{238}$$

from which the self-inductance in one arm can be determined if the other self-inductance be known together with the resistances in the two noninductive arms.

MANIPULATION. — In Fig. 123 is shown a convenient arrangement of Wheatstone bridge consisting of four noninductive resistances, the coil L whose self-inductance is sought, the variable standard of self-inductance L_s arranged so that by throwing the double-pole double-throw switch K_1, either a direct or an alternating electromotive force can be impressed on the circuit, while by use of the similar switch K_2, either a direct-current galvanometer or a vibration galvanometer can be connected into the bridge wire.

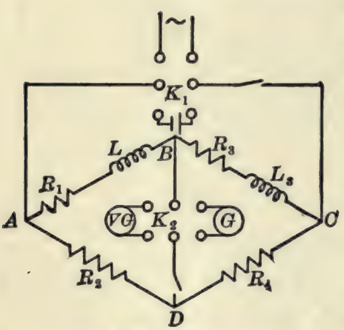

FIG. 123.

The alternating current may be obtained from a commercial 110-volt circuit if cut down to the proper value by one or more lamps in series. The direct current is supplied by one or more dry cells, the number depending upon the sensitivity of the galvanometer used.

Throw the switches so that the dry cells and the direct-current indicator are in circuit. Set the ratio arms R_2 and R_4, and adjust the resistances in the other arms until the bridge is in balance. Note the resistances R_2 and R_4.

Without changing any of the resistances, throw the switches so that the alternating current and the vibration galvanometer are in circuit. Now adjust the variable standard of self-induction L_s till a balance is again obtained. Note the reading of the inductance standard L_s. Then, from (238),

$$L = L_s \frac{R_2}{R_4}. \tag{239}$$

Exp. 105. Comparison of Mutual Inductances by the Maxwell Bridge Method

THEORY OF THE EXPERIMENT. — Read Arts. 60 and 61. If the current in one of two neighboring coils of wire be changed at the rate $\Delta i/\Delta t$, there will be induced in the other coil an electromotive force of the value

$$E = -M \frac{\Delta i}{\Delta t},$$

where M is the coefficient of mutual induction of the coils. This electromotive force will set into motion a charge (199)

$$\Delta q = -\frac{M \Delta i}{R},$$

where R is the resistance of the secondary circuit.

Suppose we have two pairs of coils of mutual inductances M_1 and M_2, and the current in each is changed at the same rate $\Delta i/\Delta t$. Also suppose that the resistances R_1 and R_2 in the secondary coils are adjusted till the discharges Δq_1 and Δq_2 in them are equal. Then, from the above, we have

$$\Delta q_1 = -\frac{M_1 \Delta i}{R_1} \tag{240}$$

and

$$\Delta q_2 = -\frac{M_2 \Delta i}{R_2}. \tag{241}$$

Dividing each member of (240) by the corresponding member of (241), we find the ratio of the mutual inductances of the two pairs of coils to be

$$\frac{M_1}{M_2} = \frac{R_1}{R_2}.$$

This principle is the basis of the Maxwell Bridge Method. In this method the primaries of the mutual inductance coils M_1 and M_2 are joined in series with a battery, ammeter, key, and rheostat. The secondaries are joined in series with variable resistances r_1 and r_2, and a ballistic galvanometer is bridged across as shown in Fig. 124. On pressing the key, the same current will traverse the two primary coils, and the galvanometer will be traversed by two charges in opposite directions — one from the secondary of M_1 and another from M_2. The resistances r_1 and r_2 are adjusted till the galvanometer throw is zero.

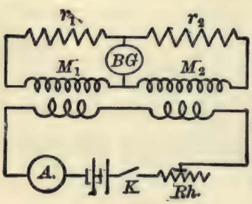

FIG. 124.

The total discharge due to M_1 is (240)

$$\Delta q_1 \left[= -M_1 \frac{\Delta i}{R_1} \right] = -\frac{M_1 \Delta i}{R' + \dfrac{gR''}{g + R''}},$$

where R' represents the sum of the resistance r_1 and that of the secondary of M_1, R'' represents the sum of r_2 and the resistance of the secondary of M_2, g is the galvanometer resistance, and M_1 is the mutual inductance of the pair of coils M_1 in Fig. 124. Part of this discharge traverses the galvanometer and the remainder traverses the shunt resistance R''.

The quantity due to M_1 discharged through the galvanometer is

$$\Delta q_1' = -\frac{M_1 \Delta i}{R' + \dfrac{gR''}{g + R''}} \cdot \frac{R''}{(R'' + g)}. \tag{242}$$

Similarly, the quantity due to M_2 discharged through the galvanometer is

$$\Delta q_2' = -\frac{M_2 \Delta i}{R'' + \dfrac{gR'}{g + R'}} \cdot \frac{R'}{(R' + g)}. \tag{243}$$

Since the experiment has been so arranged that $\Delta q_1' = \Delta q_2'$, we have, by equating the right-hand members of (242) and (243),

$$\frac{M_1 R''}{\left(R' + \dfrac{gR''}{g + R''}\right)(R'' + g)} = \frac{M_2 R'}{\left(R'' + \dfrac{gR'}{g + R'}\right)(R' + g)}.$$

Expanding this equation and cancelling, we obtain

$$\frac{M_1}{M_2} = \frac{R'}{R''}. \tag{244}$$

Manipulation. — After connecting up the apparatus, test if the discharges through the galvanometer due to M_1 and M_2 are in opposite directions. This can be done by disconnecting r_2, tapping the key, and observing the throw due to M_1; then, after reconnecting r_2, disconnecting r_1, tapping the key, and observing the throw due to M_2. Adjust the rheostat Rh till these throws are large.

Now adjust r_1 and r_2 till the galvanometer throw is zero, and note the value of r_1 and r_2. Measure the resistances of the secondary coils of M_1 and M_2. The data are now at hand for use in (244).

Exp. 106. Calibration of an Ammeter by Means of a Potentiometer

Theory of the Experiment. — Read Arts. 40, 51–53.

Manipulation. — Connect the ammeter A, Fig. 125, in series with a standard resistance R, rheostat Rh, and a battery of accumulators of sufficient number of cells to give a current that will produce a full scale ammeter deflection. Join the terminals of the standard resistance to the potentiometer.

Adjust the rheostat until the ammeter deflection is about one-

twentieth of the full scale, and by means of the potentiometer measure the potential difference $V - V_1$ at the terminals of R.

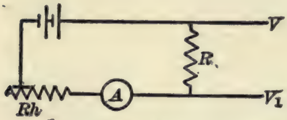

FIG. 125.

Then, the current in the ammeter is

$$I = \frac{V - V_1}{R}.$$

Note the ammeter reading and the potentiometer reading.

Take similar readings at about equal increments of current throughout the range of the ammeter.

Plot ammeter readings against the corrections to be applied.

Exp. 107. The B-H Curve for a Sample of Iron by Rowland's Method

THEORY OF THE EXPERIMENT. — Read Arts. 1, 35, 55, 56, 60 and 61. In this experiment, a specimen of iron or steel is subjected to magnetizing fields of various intensities, and the corresponding induction densities determined. Induction densities or magnetic inductions being usually represented by the symbol B, and magnetizing fields or magnetizing forces by the symbol H, the curve coördinating these quantities is called the B-H curve for the given specimen.

The value of B must be determined under conditions that either eliminate or allow for the demagnetizing action of any poles developed in the specimen. By using a specimen in the form of a closed ring, and magnetizing it axially, no poles will be developed. In Rowland's method a ring-shaped specimen is uniformly wrapped with a coil of wire joined to a source of electromotive force. In addition, there is wrapped about the ring a secondary coil whose terminals can be connected to a ballistic galvanometer. On making or breaking the current in the primary winding, a charge will be induced in the secondary coil having a value that can be obtained from the throw and known constant of the ballistic galvanometer. Knowing this quantity of charge, the induction density B within the specimen can be computed. The magnetizing field H is computed from the value of the current in the primary coil.

The ballistic constant of the galvanometer can be conveniently found by means of a standard of mutual inductance. A convenient arrangement of connections for carrying out the experiment is shown in Fig. 126.

If the magnetic flux in the direction of the axis of a coil of n turns changes by $d\Phi$ there is a change of flux-turns or linkings equal

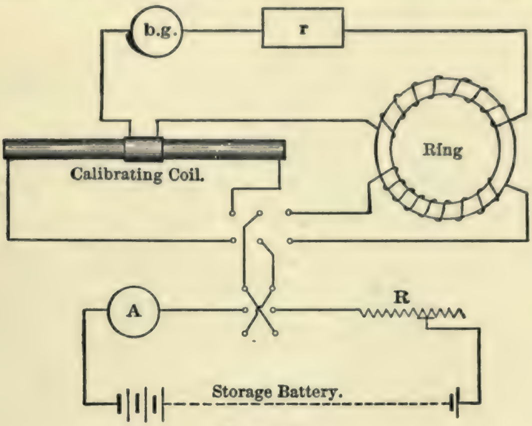

FIG. 126.

to $n\,d\Phi$ which we will represent by the symbol N. The electromotive force thereby induced in the coil is $E = -dN/dt$, and the current is $i = dq/dt$. If the total resistance of the circuit be R, we will have from Ohm's law,

$$\frac{dq}{dt} = -\frac{dN}{dt \cdot R}.$$

Whence,

$$q = -\frac{\Delta N}{R}, \tag{245}$$

or, the quantity of electricity set into motion equals the change in flux-turns divided by the resistance of the circuit.

Now consider a ring made of iron of cross section A, in which there is a change of induction density ΔB. If there be n_2 turns of wire in the secondary coil, there will be a change of flux-turns in the secondary coil

$$\Delta N\,[= n_2\Delta\Phi] = n_2 A\Delta B.$$

If the change of induction density ΔB is due to a change in the value of the induction density from $+B$ to $-B$, or the reverse, $\Delta B = 2\,B$, and we may write

$$\Delta N = 2\,n_2 AB.$$

If the total resistance in the circuit of the secondary be R', the quantity of charge q' will be, (245), (152),

$$q' = -\frac{2\,n_2 AB}{R'} = G'd', \tag{246}$$

where d' is the throw of a ballistic galvanometer produced by this charge, and G' is the constant of the instrument.

The ballistic constant of the galvanometer can be determined by connecting the galvanometer to the secondary of the calibrating coil of mutual inductance M, and noting the throw produced when the current in the primary of the calibrating coil is changed by an amount Δi. Thus, if the charge set into motion be represented by q'', (199),

$$q'' = -\frac{M\Delta i}{R''},$$

where R'' is the total resistance in circuit.

If Δi be due to a reversal of a current i, $\Delta i = 2\,i$. If, in addition, the charge q'' produces a throw d'' for a galvanometer of ballistic constant G'',

$$q'' = -\frac{2\,Mi}{R''} = G''d''. \tag{247}$$

If the same galvanometer is used for the measurement of the two discharges, and the resistances of the two secondary circuits are equal, the damping of the galvanometer will be the same in the two cases, and the ballistic constants will be equal. Whence, dividing each member of (246) by the corresponding member of (247), we have for the induction density of the specimen,

$$B = \frac{d'Mi}{d''n_2A}\text{ gausses.} \tag{248}$$

If M be expressed in henrys and i in amperes, the above equation must be written

$$B = 10^8 \frac{d'Mi}{d''n_2A} \text{ gausses.} \tag{249}$$

From (193), the intensity of the magnetizing field in the center of a long coil having n' turns per centimeter, traversed by a current of i' absolute units, is

$$H = 4\,\pi n'i' \text{ gausses.}$$

If, however, the current be expressed in amperes,

$$H = 0.4\,\pi n'i' \text{ gausses.} \tag{250}$$

To simplify the computation of a large number of values of H and B, it will be convenient to rewrite (249) and (250) in the forms

$$B = \left[\frac{10^8 M}{n_2 A}\right]\left(\frac{i}{d''}\right) d' \text{ gausses,} \tag{251}$$

$$H = [0.4\,\pi n']\, i' \text{ gausses,} \tag{252}$$

where the quantities within the brackets are constants which can be computed from the dimensions of the apparatus, and the quantity within the parenthesis is a constant which can be obtained from experiment.

In deriving these equations it has been assumed that the primary coil on the specimen uniformly covers the entire length of the ring, and that the total resistance of the circuit containing the galvanometer and the secondary of the calibrating coil equals that of the circuit containing the galvanometer and the secondary coil on the specimen. This latter requirement is most easily met by having the two secondary coils and the ballistic galvanometer in series, as shown in Fig. 126.

Manipulation. — The following constants must be known before beginning the present experiment: — the mutual inductance M of the calibrating coil, the cross section of the specimen, the total number of turns of wire n_2 in the secondary coil on the specimen, and the number of turns per unit length n' in the primary coil on the specimen.

With the primary of the calibrating coil connected to the battery, adjust the current, till on reversing the current a large galvanometer throw is obtained. Reverse the current and note the throw. Again reverse the current and note the throw. The mean of these two throws is d''. Note the ammeter reading i. Compute i/d''.

In the same manner find values of i/d'' when the current is $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ of the first value. The mean of these values of i/d'' is the value to be used in (251).

Demagnetize the specimen as follows. Arrange the double-pole double-throw switch, shown in the center of Fig. 126, so that the primary coil of the specimen is connected to the battery, and adjust the magnetizing current to such a value that it will produce a magnetizing field of force of about 30 gausses. The required current can be computed from (252). Now rock the reversing switch back and forth while the current is gradually reduced to zero by means of the rheostat R.

We are now ready to determine values of B corresponding to various values of H. On account of the fact that when the magnetizing field is changed, the change in induction density always lags behind the change in the magnetizing field which produces it, in order to obtain the correct value of B corresponding to any value of H it is necessary to reverse the direction of the magnetizing field a number of times. By this device the specimen is brought to a "cyclic state" of stable magnetic equilibrium.

Open the galvanometer circuit; connect the ring primary to the battery; adjust the current to such a value that H is not more than five gausses; reverse the current not less than 10 times. Close the galvanometer circuit, reverse the current, and note the galvanometer throw. Read the magnetizing current. The data are now at hand for computing one value of H and the value of B which this magnetizing field produces in the given specimen.

Open the galvanometer circuit. Increase the current to about double the former value and reverse the current not less than 10 times. Close the galvanometer circuit, reverse the current, and again read the galvanometer throw and the magnetizing current.

Another value of H and the corresponding value of B can now be computed.

Proceeding in the same manner, subject the specimen to a series of ascending magnetizing fields and compute the value of B corresponding to each value of H.

With magnetizing fields of force as abscissæ, and induction densities (magnetic inductions) as ordinates, construct the required B-H curve. Such a curve is shown in *ab*, Fig. 127.

Exp. 108. Determination of the Energy Dissipated in a Specimen of Iron by Hysteresis

THEORY OF THE EXPERIMENT. — Read Exp. 107. If a specimen of iron be subjected to a magnetizing field of force which increases from zero to such a value that the specimen becomes "saturated," the B-H curve will be something like *ab*, Fig. 127. If now the

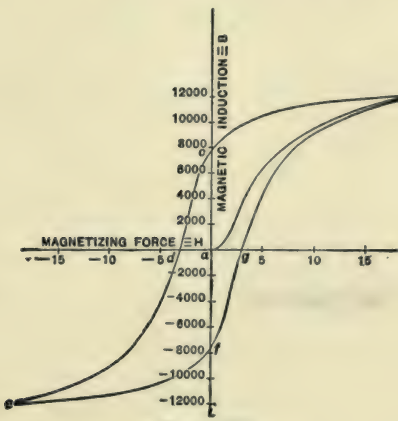

FIG. 127.

magnetizing field be diminished to zero, the relation between B and H will be as indicated by *bc*. If now the magnetizing field be reversed and increased to a negative value equal to the maximum positive value, the curve coördinating B and H will be as *cde*. And now by reversing the magnetizing field and increasing it to its original maximum value, the curve *fgb* is obtained.

An inspection of the curve shows that the change in B always lags behind the change in H. The lagging of the change in the induction density [magnetic induction] behind the magnetizing field of force is called hysteresis. The closed curve *bcdefgb* is called a *hysteresis loop.* It will now be proven that in carrying a specimen of magnetic substance through a complete cycle of magnetization, there will be an absorption of energy which is proportional to the area of the hysteresis loop for the specimen.

Consider a ring of length l and area of cross section A uniformly overwound throughout its length with a primary coil, and also overwound with a separate secondary coil.

If the current in the primary be changed there will be a change of the flux-turns within the coils. This will induce an electromotive force in the direction to oppose the change of the flux-turns of the value $E = dN/dt$. It follows that in time dt there is expended an amount of energy

$$dW\,[= Ei\,dt] = \frac{dN}{dt}\,i\,dt.$$

If the number of turns per centimeter length be denoted by n', then the total number of turns will be $n'l$. The change of flux-turns is

$$dN = n'lA \cdot dB.$$

Therefore,

$$dW = n'lAi \cdot dB.$$

And since lA is the volume of the specimen, the energy per cubic centimeter is

$$dW = n'i\,dB.$$

And since

$$H = 4\,\pi n'i,$$

$$dW = \frac{H\,dB}{4\,\pi} \text{ ergs per cc.}$$

If the magnetizing field of force varies from H_1 to H_2, the energy expended per cubic centimeter of specimen per cycle of magnetization will be

$$W = \frac{1}{4\,\pi}\int_{H_1}^{H_2} H\,dB. \tag{253}$$

Now $H\,dB$ is the area of a horizontal strip of width dB extending from the "B" axis to the B-H curve. Thus, in passing through the change of magnetization represented by ab, Fig. 127, the quantity $H\,dB$ is the area included between the "B" axis and the curve ab, and the energy expended per unit volume of specimen is measured by this area divided by 4π. In passing from b to c the energy recovered is measured by the area included between the "B" axis and the curve bc divided by 4π. Therefore, the energy dissipated is the area abc divided by 4π. Applying this method to the entire cycle of magnetization, we find that the total energy dissipated per unit volume of the specimen per cycle of magnetization equals the area of the loop $bcdefgb$ divided by 4π.

With any specimen of iron, the values of B will be so much larger than H that it will be inconvenient to plot B and H to the same scale. In this case the following method may be used to determine the proper value to take for the area of the hysteresis loop. Plot values of H along the x axis and let $H = ax$ and plot values of B along the y axis and let $B = by$, where a and b represent the number of units of H and B, respectively, corresponding to one scale division of the coördinate paper. Then we have

$$W = \frac{1}{4\pi}\int H\,dB = \frac{ab}{4\pi}\int x\,dy$$

$$= \frac{ab}{4\pi}\,(\textit{area of loop}) \text{ ergs per cc. per cycle of magnetization.} \qquad (254)$$

Since the energy dissipated is transformed into heat, we can calculate the rise in temperature of the specimen by means of the equation

Heat gained in calories = mass of 1 cc. of specimen × thermal capacity × rise in temp. + heat lost by radiation and conduction. (255)

For example, in the case of iron of density 7.7 gm. per cc., and thermal capacity 0.11 calories per gm. per ° C., if there is no heat lost by radiation or conduction, we will have, from (254) and (255),

$$\frac{\frac{1}{4\pi}\int H\,dB}{4.2 \times 10^7} = 7.7 \times 0.11 \times t.$$

Whence the rise in temperature is

$$t = \frac{\int H\,dB}{4\pi \times 4.2 \times 10^7 \times 7.7 \times 0.11} \text{ degrees centigrade.} \qquad (256)$$

MANIPULATION. — In Ewing's Fixed Point Method the magnetizing field of force and the induction density of the point b are first obtained. Then the H and the B of a series of points of the loop are determined independently of one another. By this method any reading may be repeated independently of the others, and any error in one reading does not affect any other reading.

The apparatus may be conveniently arranged as in Fig. 128. By means of the double-pole double-throw switch K_1 a current may

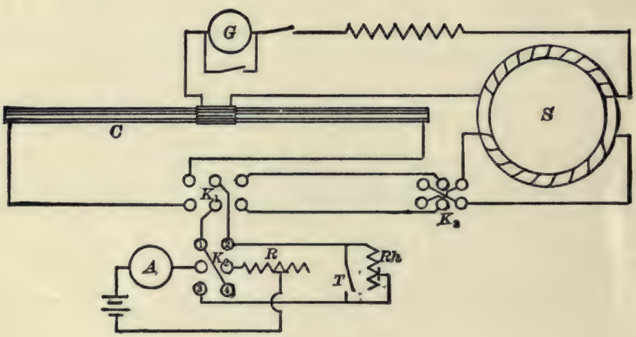

FIG. 128.

be sent through either the primary of the calibrating coil C, or the primary coil wrapped on the specimen S. K_2 is a reversing switch with one of the cross-connections removed. By means of a rheostat Rh and a short-circuiting switch T, the magnetizing field of force in one direction may be made equal to, or less than, that in the reverse direction.

Find i/d'' and demagnetize the specimen as in Exp. 107.

With a current traversing the primary coil on the specimen sufficient to produce the desired H or B value, and with T closed and the galvanometer short-circuited, rock the reversing key back and forth not less than ten times. The specimen is now in a cyclic

state. The magnetizing field and magnetic induction are as represented by the point b, Fig. 127. Put the ballistic galvanometer in circuit, again reverse the current, and note the current value and the galvanometer throw. Compute H by (252), and B by (251).

With T closed, adjust Rh until, when T is opened, the current will fall about 0.25 ampere. This will reduce the magnetizing field of force and the induction density to a new point between b and c, Fig. 127. With T closed and the galvanometer short-circuited, rock the reversing switch several times, stopping on the points 3 and 4, Fig. 128. With the galvanometer in circuit, quickly open T, and note the galvanometer throw and the current value. Compute H and the change in B. It must be kept in mind that (251) was deduced for the case where the current is reversed, and that since the change of flux-turns produced on "making" a current is one-half that produced on reversing the same current, the change of B, produced by making the current, is two times the value obtained by substituting the observed throw in (251). On subtracting this change of induction density from the previously determined value of B corresponding to the point b, Fig. 127, we can obtain the value of B for the point under consideration.

Proceeding in the same manner find the values of H and the drop of B for several points between b and c. When $H = 0$, the magnetic condition of the specimen is represented by the point c.

Up to this time the reversing switch has always been stopped on the side 3–4. Now rock the reversing switch to the side 1–2. This carries the magnetic state of the specimen to that represented by the point e. To get it back to b, reverse the primary connections on the specimen.

After rocking the reversing switch a dozen times, stopping on the side 1–2, open T. There is no throw. Adjust Rh to give small current and rock the reversing switch to the side 3–4. Note the galvanometer throw and the ammeter reading. The throw is that corresponding to a change of induction density represented by the point b to a point between c and d. Compute H and the drop in the induction density. In the same manner by increasing the magnetizing current by one-quarter ampere steps, find the coordinates of a series of points extending from c to e.

One-half of the hysteresis loop has now been determined. After this half has been plotted, the other half can be drawn by symmetry. It will be observed that the magnetic induction of *b* has been determined; whereas for each of the other points the thing that has been obtained is the difference between its magnetic induction and that of *b*. Consequently, it will be simplest to plot the coördinates of the various experimentally determined points with reference to *b* as the origin of coördinates. In this manner the curve *bcde* is constructed with any convenient scales for H and B. Now using the same coördinates, reversed in direction, and with *e* as the origin of coördinates, plot the other half of the loop *efgb*.

Measure the area of the hysteresis loop with a planimeter, and compute the energy dissipated by the specimen through hysteresis, expressed in ergs per cc. per cycle of magnetization, and also in watts per pound at 60 cycles per second.

APPENDIX

The Sum of the Series $\cos x + \cos 2x + \cos 3x \cdots$.

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

and

$$\cos(x-y) = \cos x \cos y + \sin x \sin y.$$

Hence we may write

$$\cos(n+1)x = \cos nx \cos x - \sin nx \sin x,$$
$$\cos(n-1)x = \cos nx \cos x + \sin nx \sin x.$$

Adding,

$$2\cos nx \cos x = \cos(n+1)x + \cos(n-1)x.$$

Subtracting each member of this equation from the corresponding member of $\cos nx = \cos nx$, and then adding $\cos nx$ to each member of the result,

$$2(1-\cos x)\cos nx = -\cos(n-1)x + 2\cos nx - \cos(n+1)x.$$

We will now form the series of equations obtained by substituting in the above, $n = 0$, $n = 1$, $n = 3$, etc., and add the equations:

$$\begin{aligned}
2(1-\cos x)\tfrac{1}{2} &= 1 - \cos x.\\
2(1-\cos x)\cos x &= -1 + 2\cos x - \cos 2x.\\
2(1-\cos x)\cos 2x &= -\cos x + 2\cos 2x - \cos 3x.\\
2(1-\cos x)\cos 3x &= -\cos 2x + 2\cos 3x - \cos 4x.\\
\cdots &\cdots\\
2(1-\cos x)\cos(n-1)x &= -\cos(n-2)x + 2\cos(n-1)x\\
&\quad - \cos nx.\\
2(1-\cos x)\cos nx &= -\cos(n-1)x + 2\cos nx\\
&\quad - \cos(n+1)x.
\end{aligned}$$

Adding these equations,

$$2(1-\cos x)\left(\tfrac{1}{2} + \cos x + \cos 2x + \cos 3x + \cdots\right)$$
$$= \tfrac{1}{2}[\cos nx - \cos(n+1)x],$$

or,

$$\cos x + \cos 2x + \cos 3x + \cdots$$
$$= -\frac{1}{2} + \frac{1}{2}\,\frac{\cos nx - \cos(n+1)x}{1 - \cos x}. \qquad (257)$$

The fraction can be reduced to a simpler form as follows: First consider the numerator. Suppose we put $\alpha = p + q$ and $\beta = p - q$. Adding, we obtain

$$p = \tfrac{1}{2}(\alpha + \beta). \qquad (258)$$

Subtracting, we obtain

$$q = \tfrac{1}{2}(\alpha - \beta). \qquad (259)$$

Now

$$\cos\alpha\,[= \cos(p+q)] = \cos p \cos q - \sin p \sin q$$

and

$$\cos\beta\,[= \cos(p-q)] = \cos p \cos q + \sin p \sin q.$$

Whence,

$$\cos\alpha - \cos\beta = -2\sin p \sin q.$$

Substituting in this equation the values of p and q from (258) and (259),

$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha+\beta)\sin\tfrac{1}{2}(\alpha-\beta).$$

Now set $\alpha = nx$ and $\beta = (n+1)x$. Then we have

$$\cos nx - \cos(n+1)x = -2\sin\tfrac{1}{2}(2n+1)x\sin\tfrac{1}{2}(-x)$$
$$= 2\sin(2n+1)\frac{x}{2}\sin\frac{x}{2},$$

which is the numerator of the fraction in (257).

Again, since $$\sin\frac{1}{2}x = \sqrt{\frac{1-\cos x}{2}},$$

we have for the denominator of (257)

$$1 - \cos x = 2\sin^2\frac{x}{2}.$$

Substituting for the numerator and the denominator of (257), the values just obtained, we find

$$\cos x + \cos 2x + \cos 3x + \cdots = -\frac{1}{2} + \frac{1}{2}\,\frac{\sin(2n+1)\frac{x}{2}}{\sin\frac{x}{2}}. \qquad (260)$$

TABLE 1. — THE GREEK ALPHABET

Letter	Name	Letter	Name	Letter	Name
Α, α	Alpha	Ι, ι	Iota	Ρ, ρ	Rho
Β, β	Beta	Κ, κ	Kappa	Σ, σ	Sigma
Γ, γ	Gamma	Λ, λ	Lambda	Τ, τ	Tau
Δ, δ	Delta	Μ, μ	Mu	Υ, υ	Upsilon
Ε, ε	Epsilon	Ν, ν	Nu	Φ, φ	Phi
Ζ, ζ	Zeta	Ξ, ξ	Xi	Χ, χ	Chi
Η, η	Eta	Ο, ο	Omicron	Ψ, ψ	Psi
Θ, θ	Theta	Π, π	Pi	Ω, ω	Omega

TABLE 2. — CORRECTIONS FOR THE INFLUENCE OF GRAVITY ON THE HEIGHT OF THE BAROMETER

(*a*) Reduction to Latitude 45°

From 0° to 45° the corrections are subtractive; from 45° to 90° the corrections are additive.

Lat.	Barometric height in mm. reduced to 0° C.												Lat.
	670	680	690	700	710	720	730	740	750	760	770	780	
	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	
0°	1.74	1.76	1.79	1.81	1.84	1.86	1.89	1.92	1.94	1.97	1.99	2.02	90°
5°	0.71	0.73	0.76	0.79	0.81	0.84	0.86	0.89	0.91	0.94	0.96	1.99	85°
10°	0.63	0.65	0.68	0.70	0.73	0.75	0.78	0.80	0.83	0.85	0.87	0.90	80°
15°	0.50	0.53	0.55	0.57	0.59	0.61	0.64	0.66	0.68	0.70	0.73	0.75	75°
20°	0.33	0.35	0.37	0.39	0.41	0.43	0.45	0.47	0.49	0.51	0.53	0.55	70°
25°	0.12	0.13	0.15	0.17	0.18	0.20	0.22	0.23	0.25	0.27	0.28	0.30	65°
30°	0.87	0.88	0.89	0.91	0.92	0.93	0.95	0.96	0.97	0.98	0.00	0.01	60°
35°	0.59	0.60	0.61	0.62	0.63	0.64	0.65	0.66	0.66	0.67	0.68	0.69	55°
40°	0.30	0.31	0.31	0.31	0.32	0.32	0.33	0.33	0.34	0.34	0.35	0.35	50°
45°	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	45°

(*b*) Reduction to Sea Level

Corrections are subtractive

Elevation	Barometric height in mm. reduced to 0° C.						
	660	680	700	720	740	760	770
m.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
100	....	....	0.01	0.01	0.01	0.01	0.02
200	....	0.03	0.03	0.03	0.03	0.03	0.03
300	....	0.04	0.04	0.04	0.04	0.04	......
400	0.05	0.05	0.05	0.06	0.06	0.06	......
500	0.06	0.07	0.07	0.07	0.07	0.07	......
600	0.08	0.08	0.08	0.08	0.09	......	......
700	0.09	0.09	0.10	0.10	0.10	......	......
800	0.10	0.11	0.11	0.11	0.12	......	......
900	0.12	0.12	0.12	0.13	....	......	....
1000	0.13	0.13	0.14	0.14	....	......	......

TABLE 3. — BOILING POINT OF WATER UNDER DIFFERENT BAROMETRIC PRESSURES

(a) Temperatures in Degrees Centigrade and Pressures in Millimeters of Mercury

°C.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
90	525.4	527.4	529.4	531.4	533.4	535.5	537.5	539.6	541.6	543.7
91	545.7	547.8	549.9	551.9	554.0	556.1	558.2	560.3	562.4	564.6
92	566.7	568.8	571.0	573.1	575.3	577.4	579.6	581.8	584.0	586.1
93	588.3	590.5	592.7	595.0	597.2	599.4	601.6	603.9	606.1	608.4
94	610.7	612.9	615.2	617.5	619.8	622.1	624.4	626.7	629.0	631.4
95	633.7	636.0	638.4	640.7	643.1	645.5	647.9	650.2	652.6	655.0
96	657.4	659.9	662.3	664.7	667.1	669.6	672.0	674.5	677.0	679.4
97	681.9	684.4	686.9	689.4	691.9	694.5	697.0	699.5	702.1	704.6
98	707.2	709.7	712.3	714.9	717.5	720.1	722.7	725.3	727.9	730.5
99	733.2	735.8	738.5	741.2	743.8	746.5	749.2	751.9	754.6	757.3
100	760.0	762.7	765.5	768.2	770.9	773.7	776.5	779.2	782.0	784.8

(b) Temperatures in Degrees Fahrenheit and Pressures in Inches of Mercury

°F.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
194	20.68	20.73	20.77	20.82	20.86	20.90	20.95	29.99	21.04	21.08
195	21.13	21.17	21.22	21.26	21.30	21.35	21.39	21.44	21.48	21.53
196	21.58	21.62	21.67	21.71	21.76	21.80	21.85	21.89	21.94	21.99
197	22.03	22.08	22.12	22.17	22.22	22.26	22.31	22.36	22.40	22.45
198	22.50	22.54	22.59	22.64	22.69	22.73	22.78	22.83	22.88	22.92
199	22.97	23.02	23.07	23.11	23.16	23.21	23.26	23.31	23.36	23.40
200	23.45	23.50	23.55	23.60	23.65	23.70	23.75	23.80	23.85	23.89
201	23.94	23.99	24.04	24.09	24.14	24.19	24.24	24.29	24.34	24.39
202	24.44	24.49	24.54	24.59	24.64	24.69	24.74	24.80	24.85	24.90
203	24.95	25.00	25.05	25.10	25.15	25.21	25.26	25.31	25.36	25.41
204	25.46	25.52	25.57	25.62	25.67	25.73	25.78	25.83	25.88	25.94
205	25.99	26.04	26.10	26.15	26.20	26.25	26.31	26.36	26.42	26.47
206	26.52	26.58	26.63	26.68	26.74	26.79	26.85	26.90	26.96	27.01
207	27.07	27.12	27.18	27.23	27.29	27.34	27.40	27.45	27.51	27.56
208	27.62	27.67	27.73	27.79	27.84	27.90	27.95	28.01	28.07	28.12
209	28.18	28.24	28.29	28.35	28.41	28.46	28.52	28.58	28.64	28.69
210	28.75	28.81	28.87	28.92	28.98	29.04	29.10	29.16	29.21	29.27
211	29.33	29.39	29.45	29.51	29.57	29.62	29.68	29.74	29.80	29.86
212	29.92	29.98	30.04	30.10	30.16	30.22	30.28	30.34	30.40	30.46

TABLE 4. — PRESSURE OF SATURATED AQUEOUS VAPOR

In Millimeters of Mercury

° C.	Pressure	° C.	Pressure	° C.	Pressure	° C.	Pressure	° C.	Pressure
1	4.91	31	33.37	61	155.95	91	545.77	121	1539.25
2	5.27	32	35.32	62	163.29	92	566.71	122	1588.47
3	5.66	33	37.37	63	170.92	93	588.33	123	1638.96
4	6.07	34	39.52	64	178.86	94	610.64	124	1690.76
5	6.51	35	41.78	65	187.10	95	633.66	125	1743.88
6	6.97	36	44.16	66	195.67	96	657.40	126	1798.35
7	7.47	37	46.65	67	204.56	97	681.88	127	1854.20
8	7.99	38	49.26	68	213.79	98	707.13	128	1911.47
9	8.55	39	52.00	69	223.37	99	733.16	129	1970.15
10	9.14	40	54.87	70	233.31	100	760.00	130	2030.28
11	9.77	41	57.87	71	243.62	101	787.59	131	2091.94
12	10.43	42	61.02	72	254.30	102	816.01	132	2155.03
13	11.14	43	64.31	73	265.38	103	845.28	133	2219.69
14	11.88	44	67.76	74	276.87	104	875.41	134	2285.92
15	12.67	45	71.36	75	288.76	105	906.41	135	2353.73
16	13.51	46	75.13	76	301.09	106	938.31	136	2423.16
17	14.40	47	79.07	77	313.85	101	971.14	137	2494.23
18	15.33	48	83.19	78	327.05	108	1004.91	138	2567.00
19	16.32	49	87.49	79	340.73	109	1039.65	139	2641.44
20	17.36	50	91.98	80	354.87	110	1075.37	140	2717.63
21	18.47	51	96.66	81	369.51	111	1112.09	141	2795.57
22	19.63	52	101.55	82	384.64	112	1149.83	142	2875.30
23	20.86	53	106.65	83	400.29	113	1188.61	143	2956.86
24	22.15	54	111.97	84	416.47	114	1228.47	144	3040.26
25	23.52	55	117.52	85	433.19	115	1269.41	145	3125.55
26	24.96	56	123.29	86	450.47	116	1311.47	146	3212.74
27	26.47	57	129.31	87	486.32	117	1354.60	147	3301.87
28	28.07	58	135.58	88	486.76	118	1399.02	148	3392.98
29	29.74	59	142.10	89	505.81	119	1444.55	149	3486.09
30	31.51	60	148.88	90	525.47	120	1491.28	150	3581.23

TABLE 5. — PRESSURE OF SATURATED MERCURY VAPOR

In Millimeters of Mercury

° C.	Pressure	° C.	Pressure	° C.	Pressure	° C.	Pressure	° C.	Pressure
0	0.00047	10	0.00080	20	0.00133	70	0.050	120	0.779
2	0.00052	12	0.00089	30	0.0029	80	0.093	130	1.24
4	0.00058	14	0.00099	40	0.0063	90	0.165	140	1.93
6	0.00064	16	0.00109	50	0.013	100	0.285	150	2.93
8	0.00072	18	0.00121	60	0.026	110	0.478	160	4.38

TABLE 6. — COEFFICIENTS OF LINEAR EXPANSION OF SOLIDS

Substance	Temp., ° C.	α	Substance	Temp., ° C.	α
Aluminium...	40	0.000023	Iron (soft)....	40	0.000012
Brass.........	0 to 100	0.000019	Iron (cast)....	40	0.000011
Copper.......	40	0.000017	Lead.........	40	0.000029
German silver	0 to 100	0.000018	Nickel........	40	0.000013
Glass (crown).	0 to 100	0.000008	Silver........	40	0.000019
Glass (flint)..	0 to 100	0.000009	Zinc..........	40	0.000029

TABLE 7. — COEFFICIENTS OF CUBICAL EXPANSION OF LIQUIDS

$$V_t = V_0 (1 + \beta_1 t + \beta_2 t^2 + \beta^3 t^3)$$

Substance	Temperature	β_1	β_2	β_3
Alcohol *	−39 to 27° C.	0.001033	0.00000145	....
	27 to 46	0.001012	0.00000220	....
Analin...	7 to 154	0.000817	0.00000092	0.000000000628
Glycerin.	....	0.000485	0.00000049	....
Mercury.	24 to 299	0.0001818	0.00000000018	0.00000000035
Water...	0 to 25	−0.00006106	0.000007718	−0.00000003734
	25 to 50	−0.00006542	0.000007759	−0.00000003541
	50 to 75	−0.00005916	0.000003185	0.00000000728
	75 to 100	−0.00008645	0.000003189	0.00000000245

* 93.3% (by volume) pure.

TABLE 8. — HEAT VALUES OF VARIOUS FUELS

h indicates the number of gram calories of heat developed by the complete oxidation of one gram of substance. h' indicates the number of gram calories developed by the burning of one liter of gas measured at 0° C. and 760 mm. pressure. If water is one of the products of combustion, the heat value is given when the water is in the liquid form.

Substance	h	Substance	h'
Cane Sugar............	3866	Acetylene..............	14460
Carbon (charcoal)......	8080	Benzene vapor..........	33496
Cellulose..............	4140	Coal gas...............	1000–1400
Coal...................	5500–9000	Dawson gas............	5500–7000
Naphthalin............	9692	Hydrogen.............	3090
Petroleum.............	10200–11500	Natural gas (Ind.).....	9500
Peat...................	4000–4500	Water gas.............	2000–3500
Wood *................	4000–5000	(carbureted)	3500–7000

* Containing 10–12% of moisture.

TABLE 9. — SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Unless otherwise stated, the following values express the mean specific heats from 0° to 100° C.

Substance	Sp. heat	Substance	Sp. heat
Aluminium	0.219	Lead	0.032
Alcohol, ethyl	0.685	Mercury	0.033
Antimony	0.050	Marble	0.216
Bismuth	0.030	Nickel	0.113
Brass	0.093	Paraffin (solid 0°–40°)	0.560
Calcium sulphate	0.250	(liquid 50°–100°)	0.710
Copper	0.093	Platinum	0.033
Copper sulphate	0.316	Rock salt	0.219
German silver	0.095	Sandstone	0.224
Glycerin (15°–50° C.)	0.576	Silver	0.056
Glass (crown)	0.161	Sugar	0.304
Glass (flint)	0.117	Turpentine	0.467
Granite	0.193	Tin	0.056
Iron (wrought)	0.108	Vulcanite	0.331
Iron (steel)	0.117	Zinc	0.094

TABLE 10. — MELTING POINTS AND HEAT EQUIVALENTS OF FUSION

Substance	Melting point	Heat equiv. of fusion	Substance	Melting point	Heat equiv. of fusion
	° C.	Cal. per g.		° C.	Cal. per g.
Beeswax	61.8	42.3	Mercury	−29	2.8
Benzol	5.4	30	Naphthalin	79.9	35.7
Bismuth	266.8	12.6	Nickel	1450	4.6
Bromine	−7.3	16.2	Palladium	1500	36.3
Cadmium	320.7	13.7	Paraffin	52.4	35.1
Glycerin	13	42.5	Phenol	25.4	24.9
Ice	0.0	80	Platinum	1779	27.2
Iodine	115	11.7	Silver	999	21.1
Iron, cast (gray)	1200	23	Sulphur	115	9.4
Iron, cast (white)	1100	33	Tin	233	14.3
Lead	326	5.4	Zinc	415	28.1

TABLE 11. — BOILING POINTS AND HEAT EQUIVALENTS OF VAPORIZATION

Substance	Boiling point	Heat equiv. of vap.	Substance	Boiling point	Heat equiv. of vap.
	° C.	Cal. per g.		° C.	Cal. per g.
Acetic acid......	188	84.9	Benzol.......	79.6	93.5
Acetone.........	56.6	125.3	Chloroform...	61	58.5
Analin..........	182.5	93.3	Ether, ethyl..	35	90.5
Alcohol, ethyl...	78	205	Mercury......	350	62
Alcohol, methyl..	64.5	267.5	Water........	100	536

INDEX TO VOL. II